IN FACULTY OF ENGINEERING

Application of Topology Optimization and 3D Printing in the Construction Industry

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Gieljan Vantyghem

Doctoral dissertation submitted to obtain the academic degree of Doctor of Civil Engineering Technology

Supervisors

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DECLARATION

I declare that this doctoral thesis is an original report of my research, has been written by me and has not been submitted for any previous degree or professional qualification. The collaborative contributions have been indicated clearly and acknowledged. Due references have been provided on all supporting literatures and resources.

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Date: June 2021

Gieljan Vantyghem, MSc

Ghent, Belgium

To Sarah and Rosie

ABSTRACT (DUTCH)

Stap één in het verduurzamen van het bouwproces is het hebben van een duurzaam ontwerp. Dit doctoraatsonderzoek tracht het ontwerpproces te optimaliseren door het bestuderen van bestaande en het ontwikkelen van vernieuwende wiskundige ontwerptechnieken. De basisalgoritmen die hiervoor gebruikt worden zijn grotendeels gebaseerd op vorm- en topologie-optimalisatiemethoden. In dit werk worden de algoritmes theoretisch uitgewerkt, maar ook hun toepassingen en voordelen ten opzichte van de klassieke ontwerpmethoden in een bouwkundig ontwerpproces worden gedemonstreerd. Een waaier aan onontgonnen architecturale mogelijkheden wordt hierdoor geopend. Hierbij wordt rekening gehouden met prille, maar veelbelovende, productiemethoden die opkomen in de bouwsector, zoals 3D (beton)printen.

Het ontwerp van gebouwschilonderdelen en -systemen is een proces waar vele verschillende aspecten (structurele prestaties, bouwfysische eigenschappen, duurzaamheid, etc.) aan te pas komen. De gebouwschil vormt namelijk de fysieke scheiding tussen de binnen- en buitenomgeving. In vele gevallen worden deze vakspecifieke disciplines nog te veel afzonderlijk beschouwd. Hierbij worden bijvoorbeeld de structurele prestaties geoptimaliseerd, en worden de thermische eisen pas in tweede fase bekeken. Een benadering waarbij deze aspecten tijdens het ontwerp gelijktijdig zouden worden geoptimaliseerd, zou een grote meerwaarde kunnen betekenen. Dit zou de snelheid van het ontwerpproces en tegelijkertijd de efficiëntie van het onderdeel kunnen verhogen.

In een eerste onderzoeksluik wordt het belang van zo'n multidisciplinaire aanpak bij het ontwerp van gebouwschilonderdelen onderzocht. Het doel is een aantal casestudies te verzamelen waar een gelijktijdige structurele en thermische optimalisatie het ontwerp vooruit kunnen helpen. Binnen dit thema wordt het bestaande denkkader deels doorbroken, en de creatieve mogelijkheden binnen het ontwerp worden verkend. Bestaande algoritmes worden geëxploreerd en vernieuwende (gecombineerde) algoritmes worden ontwikkeld. Iedere casestudie bouwt ook verder op de bevindingen, maar ook op de beperkingen van de vorige.

Een eerste casestudie die in deze thesis uitvoerig wordt besproken, bestudeert de invloed die topologie-optimalisatietechnieken kunnen hebben op het ontwerp van isolerend metselwerk. Structurele topologieoptimalisatie is de meest algemene vorm van structurele optimalisatie waarbij een wiskundig algoritme kan bepalen waar (binnen een voorafgedefinieerd ontwerpdomein) beter wel, en waar beter geen, materiaal geplaatst wordt. Binnen deze casestudie, wordt de invloed van de vorm van de interne holtes in de snelbouwstenen geanalyseerd en topologie-optimalisatie wordt gebruikt om nieuwe vormen op een autonome wijze te ontdekken. Bestaande algoritmes worden uitgebreid, aangepast, en samengevoegd, en een nieuwe gecombineerde ontwerpmethodiek wordt op punt gezet. Het resultaat van deze eerste studie is een reeks geoptimaliseerde bouwstenen met verbeterde structurele én thermische prestaties. De ontwikkelde tools maken het ook mogelijk om af te wijken van de traditionele ontwerpmethodiek, waar de nadruk vaak ligt op een reeks individuele disciplinaire simulaties en/of optimalisaties. In de voorgestelde nieuwe methode, worden de optimale vormen bepaald met de grootst mogelijke ontwerpvrijheid, en aan de hand van duidelijk gedefinieerde randvoorwaarden en doelstellingen.

Naast de vele voordelen die de voorgestelde methodiek met zich meebrengt, zijn er ook een aantal beperkingen die komen boven drijven. Een probleem dat zich kenbaar maakt, is dat bepaalde parameters, zoals de materiaal-specifieke interpolatiecurves, een grote invloed hebben op het resultaat. Zo'n interpolatiecurve legt het verband vast tussen de ontwerpvariabelen van het optimalisatieprobleem (de densiteit van de FE-mesh elementen) en materiaalkarakteristieken, zoals de elasticiteitsmodulus en de thermische geleiding van het materiaal. In bepaalde situaties wordt bijvoorbeeld geen oplossing gevonden of is het resultaat niet duidelijk af te lezen. De ontwerpvariabelen zijn dan vaak niet geconvergeerd naar een zwart-wit topologie (en bevat dus veel grijswaarden). Een correcte interpretatie van dit resultaat (en hoe die kan worden omgezet in een fysisch object) is dan zeer moeilijk. Een tweede casestudie gaat daarom dieper in op deze problematiek en tracht om de correcte fysische relatie tussen de interpolatiecurves te vinden en ervoor te zorgen dat convergentie van het probleem beter wordt afgedwongen. De 'penalisatie'-parameters worden zodanig gekozen dat er geen tussenliggende waarden (grijswaarden) kunnen worden bekomen. Ter compensatie wordt het gebruik van een extra materiaal toegankelijk gemaakt om het optimalisatieprobleem extra vrijheid te kunnen bieden. Een multi-materialen topologieoptimalisatie script wordt uitgewerkt waarbij opnieuw de structurele en thermische prestaties gezamenlijk worden opgenomen. In de casestudie die hieraan is gelinkt, wordt vervolgens een console voor metselwerk met thermische onderbreking geoptimaliseerd. Het doel is om voor gelijke gewichtsbelasting van de console, het resultaat is een console die zowel economisch als energetisch aantrekkelijk is. Thermische onderbrekingen worden door het algoritme zelf voorzien, en de meest optimale locatie ervan wordt aangeduid. Een parameterstudie toont ook de invloed van de verschillende materiaalkarakteristieken, en parameters.

Een tweede luik binnen dit doctoraatsonderzoek probeert de technologie die goed gekend is onder de verzamelterm: '3D printing' te koppelen aan de ontwerp-optimalisatiemethoden die eerder werden besproken. Een duidelijke link die vaak gelegd wordt, is dat topologieoptimalisatietechnieken vaak zeer complexe eindresultaten produceren. En door die complexiteit zijn de traditionele productiemethoden niet meer in staat om de geoptimaliseerde structuren en objecten op een duurzame manier te maken. Het gebruik van topologie-optimalisatie is dus pas sinds de opkomst van de 3D-print technologie interessant geworden. Een ander aspect, waar in dit werk aanvullend op gefocust wordt, is dat de interpolatiecurves van een topologieoptimalisatie probleem rechtstreeks gekoppeld kunnen worden aan de invulpatronen van de 3D-print techniek. Variabele dichtheden (de ontwerpvariabelen van het optimalisatieprobleem) kunnen namelijk met grote eenvoud fysisch gerealiseerd worden door de 3D printer. Dit kan door ofwel het invulpatroon of de dichtheid ervan aan te passen. In dit doctoraatswerk wordt een methodiek opgesteld om invulpatronen te karakteriseren en een derde casestudie illustreert de werking en functionaliteit ervan indien die gekoppeld worden met een multidisciplinaire topologie-optimalisatiestudie. De structurele en thermische prestaties van een driehoekig invulpatroon worden aan de hand van inverse homogenisatie-technieken bepaald, en de resultaten worden gebruikt voor het bepalen van de optimale materiaalverdeling. De casestudie waarop deze strategie is toegepast is een te printen structureel element van een dakconstructie. De volledige convergentie van het eindresultaat (de zwart-wit oplossing) is niet langer een noodzaak, omdat een realiseerbare link beschikbaar is tussen de verkregen materiaalverdeling en de fysische interpretatie ervan. Het resultaat is een complexe mengeling van verschillende invulpatronen en optimale dichtheden.

Een moeilijkheid binnen de karakterisering van de invulpatronen situeert zich in het bepalen van de invloed van thermische straling en convectie. In de vorige hoofdstukken werd enkel warmteoverdracht door geleiding bestudeerd omdat de isolerende delen van het ontwerp steeds bestonden uit een geslotencellig isolatiemateriaal waar de invloed van straling en convectie kon worden verwaarloosd. Echter, in het geval van 3D-geprinte invulpatronen, is het de stilstaande lucht in de holtes van de geprinte structuur die zorgt voor de isolerende werking. Wanneer die holtes groter worden of in hun geheel verdwijnen, dan zal een warmtestroming kunnen ontstaan die niet kan worden verwaarloosd. De verkregen methodiek houdt hier dus rekening mee en de invloed ervan op het ontwerp wordt kenbaar gemaakt via verschillende voorbeelden en casestudies.

3D printen kan met kunststof, maar er kan ook met andere materialen geprint worden. Tot nu toe gingen we ervan uit dat de 3D-print techniek, die traditioneel gebruik maakt van kunststof als printmateriaal, ook op extra grote schaal kan worden toegepast. Hoewel hiervan verschillende voorbeelden te vinden zijn, zijn het huidig marktaandeel en de toekomstperspectieven van de methode beperkt. Effectiever blijkt het 3D printen met cement-gebaseerde materialen zoals beton en mortel. Verwacht wordt dat deze techniek van 3D printen in de toekomst steeds interessanter zal worden naarmate ook voor andere bouwactiviteiten de aanwezigheid van computergestuurde kranen en robotica zal toenemen. Dit komt omdat de stap dan kleiner wordt om mits aanpassing van deze systemen sturing mogelijk te maken van betonprinters (op de werf).

De potentiële win-win situatie die ontstaat door 3D printen met beton en topologischgeoptimaliseerd betonstructuurontwerp te koppelen met elkaar, wordt aangetoond in een volgend hoofdstuk. In dit hoofdstuk wordt topologie-optimalisatie als tool gebruikt om de vorm en het ontwerp van een voorgespannen betonnen brug te optimaliseren. Enerzijds wordt het materiaalverbruik van de oorspronkelijke balkvorm gereduceerd tot het absolute minimum, en anderzijds wordt ook het traject en de positie van de naspanstreng geoptimaliseerd. De uitwerking van het specifieke algoritme voor deze optimalisatie werd weliswaar ontwikkeld aan Technion - Israel Institute of Technology, maar dit hoofdstuk beschrijft verder wel het volledig ontwerpproces, de structurele voorstudie, alle aspecten van de uitvoering, en ook een structurele validatie aan de hand van digitale beeldcorrelatie. De structurele prestaties van de geoptimaliseerde ligger worden vergeleken met een traditionele ligger en de verwachte materiaalwinst werd ingeschat op 20%. Hoewel het niet de allereerste 3D-geprinte betonconstructie was, is het wel de eerste demonstratie van hoe topologisch ontwerp in combinatie met 3D printen met beton het mogelijk maakt om efficiënte structuren te creëren. Dit laatste voorbeeld is dan ook het ideaal sluitstuk van deze thesis waar alle verschillende aspecten nog eens mooi samenkomen.

ABSTRACT (ENGLISH)

The construction industry is a massive industry that has a profoundly negative impact on our environment. An important step towards making the construction industry more sustainable is having a more sustainable design. This doctoral thesis tries to optimize the design process of construction-related components by studying existing and developing innovative mathematical design techniques. The basic algorithms used for this are based on shape and topology optimization methods. In this work, the developed frameworks are elaborated theoretically but also their applications and advantages over the classical design methods for construction-related components are demonstrated. A world of unexplored architectural designs opportunities is opened. Additionally, this research project also takes into account recent, but promising, production methods that are emerging in the construction sector, such as 3D (concrete) printing.

The design of building envelope components is a process involving many different disciplines (structural performance, building physics, durability, etc.) as the building envelope forms the physical separator between the indoor and outdoor environment. In many cases, this broad range of disciplines is still considered too much individual. For example, first the structural performances are optimized, whereafter the thermal requirements are only considered in a second phase. An approach where all these aspects are optimized in parallel could be beneficial. This could not only increase the efficiency of these components but also the overall design speed.

In a first part of this doctoral study, the importance of such a multidisciplinary framework for the design of building envelope components is investigated. The aim is to collect several case studies where a simultaneous structural and thermal optimization approach can benefit the design. Within this section, the existing framework is partly broken up, and the creative possibilities within the new framework are explored. Existing algorithms are investigated, and innovative (new and combined) algorithms are developed. Each case study also builds on what was discovered by the previous one, including its limitations. A first case study, discussed in detail in this thesis, examines the influence that topology optimization techniques can have on the design of insulating masonry blocks. Structural topology optimization is the most general form of structural optimization in which a mathematical algorithm can determine where (within a predefined design domain) material should (and should not) be placed. Within this case study, the influence of the shape of the internal cavities is analyzed and topology optimization is used to discover new shapes in an autonomous way. Existing algorithms are expanded, adapted and merged, and a new combined design methodology is constructed. The result of this first study is a series of optimized building blocks with improved thermal insulating performances. The developed tools also make it possible to deviate from the traditional design methodology, where the emphasis is often on a series of individual disciplinary simulations and/or optimizations. In the proposed new method, the optimal shapes are determined with the greatest possible design freedom and based on clearly defined preconditions and objectives.

In addition to the many advantages of the proposed method, there are also a number of limitations that emerge. One problem that stands out, is that certain parameters, such as the material-specific interpolation curves, have a large influence on the final result. Such an interpolation curve links the design variables of the optimization problem (the densities of the FE-mesh) with the material characteristics, such as the elastic modulus and thermal conductivity of the material. In certain situations, for example, the convergence of the solution is not achieved, and the result cannot be interpreted clearly. Very often, intermediate values for the design variables remain present in the design and an accurate understanding of the results is then very difficult.

A second case study therefore takes a closer look at this problem and tries to improve the physical relationship between the interpolation curves in order to better enforce the convergence of the problem. The penalization parameters are chosen so that no intermediate values can be obtained. To compensate, the use of an extra material is made accessible to offer the optimization problem extra freedom. A multi-material topology optimization script is developed, again incorporating the structural and thermal

performances. In the case study associated with this, a console for masonry with thermal breaks is optimized. The aim is to limit the heat loss through the console and optimize the material consumption for equal loading conditions on the console. The result is a console that is economically and energetically attractive. Thermal breaks are introduced by the algorithm itself, and their most optimal location is predicted. A parameter study also shows the influence of the different material characteristics and parameters.

A second part of this doctoral thesis aims to link the technology that is well known under the collective term: "3D printing", to the design optimization methods discussed earlier. A clear link that is often made, is that topology optimization techniques often have very complex end results. And because of that complexity, traditional production methods are no longer able to make these optimized structures and objects in a sustainable way. So, the use of topology optimization has only become interesting again since the emergence of 3D printing technology. Another aspect, which is additionally focused on in this work. is that the interpolation curves of a topology optimization problem can be directly linked to the infill patterns of 3D printing technology. Namely, the variable densities of the optimization problem can be physically realized by the 3D printer with great simplicity. This can be done by adjusting either the infill pattern or its density. In this doctoral work, a methodology is therefore developed to characterize 3D-printable infill patterns, and a third case study illustrates its application and functionality when coupled with a multidisciplinary topology optimization study. The structural and thermal performances of a triangular infill pattern are determined by numerical homogenization techniques, and the results are used to determine the optimized material distribution. The case study to which this strategy has been applied, is a printable structural element of a roof structure. The complete convergence of the final result (the black and white solution) is no longer a necessity, because a feasible link is available between the obtained density distribution and its physical interpretation. The result is a complex mix of different infill densities and/or patterns.

A difficulty within the characterization of the infill patterns lies in the determination of the influence of thermal radiation and convection. In the previous chapters, only heat transfer

by conduction was studied because the insulating parts of the design consisted of a known closed-cell insulating material where the influence of radiation and convection can be neglected. However, in the case of 3D printed infill patterns, it is the stagnant air in the cavities of the printed structure that provides the insulating effect. When those cavities get bigger or disappear altogether, a heat flow can arise that cannot be neglected. The method obtained takes this into account and its influence on the optimized design is demonstrated through various examples and case studies.

3D printing can be done with plastic, but it is also possible to print with other materials. Until now, we assumed that the 3D printing technique, which traditionally uses plastic as a printing material, can also be applied on a large scale. Although several examples can be found, the current market share and the prospects of the method are limited. More effective is the 3D printing with cement-based materials such as concrete and mortar. It is expected that this technique of 3D printing will become increasingly interesting in the future as the presence of computer-controlled cranes and robotics will also increase for other construction activities. This is because by adapting these systems, the step to enable control of concrete printers (on site) is reduced.

The potential win-win situation that arises by connecting 3D printing with concrete and topologically optimized concrete structures with each other is shown in a next chapter. In this chapter, topology optimization is used as a tool to optimize the shape and design of a prestressed concrete bridge. On the one hand, the material consumption of the original beam shape is reduced to the absolute minimum, and on the other hand, the trajectory and position of the post-tensioning strand is also optimized. Although the elaboration of the specific algorithm for this optimization was developed at Technion – Israel Institute of Technology, this chapter describes the complete design process, the structural preliminary study, all aspects of the implementation, as well as a structural validation based on digital image correlation. The structural performance of the optimized beam is compared with a traditional beam and the expected material gain was estimated at 20%. Although it was not the very first 3D printed concrete structure, it is the first demonstration of how topological design combined with 3D printing with concrete makes it possible to

create efficient concrete structures. This last example is therefore an ideal closing example and serves as a good ending of this doctoral thesis. A final case study where all the different aspects discussed in this thesis come together nicely.

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Dedicated to my loving family and parents: Sarah & Rosie, Tom & Evelien.

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LIST OF ABBREVIATIONS AND ACRONYMS

3D	Three Dimensional
3DP	Three-Dimensional Printing
3DCP	Three-Dimensional Concrete Printing
ABS	Acrylonitrile Butadiene Styrene (ABS plastic is a commonly used material in 3D printing)
AM	Additive Manufacturing
BIM	Building Information Modelling
CAE	Computer Aided Engineering
CNC	Computer Numerical Control
DIC	Digital Image Correlation
EPBD	Energy Performance of Buildings Directive
HS	Hashin–Shtrikman (bounds)
IP	Intellectual Property
ISE	Isotropic Solid or Empty
MDO	Multi-Disciplinary Optimization
MMA	Method of Moving Asymptotes
RAMP	Rational Approximation of Material Properties
SIMP	Solid Isotropic Microstructure with Penalization
ТО	Topology Optimization
UHPC	Ultra-High-Performance Concrete

CHAPTER I

INTRODUCTION

1.1 Context

Three-dimensional (3D) printing is a technology that has received a lot of media attention in the past decade. It is a new and challenging technology that appeals to many and makes people dream about the future. 3D printing (3DP) is a type of 'additive' manufacturing (AM) that differs from other production methods (such as CNC and traditional assembly line production) because the complete product is built 'from the ground up', in a way so that, in theory, no excessive material is used; nothing goes to waste. Where traditional systems require more standardization, and force the use of linear elements in design, the additive process of 3DP has many advantages (and its possibilities seemed limitless for some time).

Outside construction, many industries like automotive, medicine, and aerospace industry have already uncovered their full potential and found great applications that benefit from 3DP [1,2]. For example, in the automotive industry, 3DP can be used to make molds and thermoforming tools, rapid manufacturing of grips, jigs, and fixtures. This replaces expensive and long lead-time CNC productions and are often cheaper and have a shorter production time [3,4]. For medical applications, 3D printers can be used to manufacture a variety of devices, including those with complex geometry and features that match a patient's unique anatomy [5,6]. In the aerospace industry, on the other hand, 3DP is used for producing lighter parts while maintaining strength, addressing challenges like reducing an aircraft's fuel consumption by reducing its overall weight [7,8]. Additionally, for both the automotive and aerospace industry, in-house prototyping helps to control intellectual property (IP) or prevent information leaks because more components can be produced on-site [9].

For this PhD study, the fact that 3DP, and AM in general, can handle very complex geometries is the most interesting characteristic, because it is no longer the manufacturing process that determines the complexity of a product, but rather the product's desired functionality and design, without having too many annoying manufacturing constraints. Hence, we see the rebirth of certain important design optimization methods.

One of the most popular conceptual design optimization methods that has come back into the spotlight, thanks to 3DP, is topology optimization (TO). It is a mathematical principle that has been around for quite some time [10] and allows for the exploration of design by finding an optimized material distribution for any system. Optimized topologies tend to have a high complexity which hindered its usage in the past. Typically, the resulting optimized designs were nearly impossible to manufacture with conventional technologies (like milling, or casting) or were only possible at disproportionately high costs. As such, since the emergence of 3DP, they have become interesting again.

Today, many relatively easy to use and very accessible software packages have implemented a basic TO algorithm. One frequent implementation is called "minimum compliance optimization" and can be programmed in a very efficient manner [11]. TO with minimum compliance as objective, looks for a structure with the best balance of structural performances in term of stiffness and material consumption. In simple terms, the algorithm removes or displaces material within a certain design domain to make the design more efficient. A disadvantage of TO is that it only considers the predefined objective and/or its constraint functions. For example, a design that is optimized for stiffness has not necessarily a good stress distribution (or has no limitations on maximum allowable stresses). Additionally, an optimized result might behave unexpectedly under slightly different environmental conditions. As such, a real-world component that is mechanically optimized might be exposed to more than structural loads only. In such case, a multi-disciplinary approach can be considered.

Multi-disciplinary design optimization (MDO) refers to the area of research concerned with the design of complex engineering systems governed by multiple (possibly interacting) physical phenomena. This type of optimization emerged in the 1980s following the success of the application of numerical optimization techniques to structural design in the 1970s [12,13]. Aircraft design was one of the first inspiring applications of MDO. For example, Boeing's 'blended-wing-body' aircraft concept considered aerodynamics, structural analysis, propulsion, control theory, and economics collectively in order to find the optimal shape of the wing's body [14]. These days, MDO is used in many other

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industries. However, the construction sector, which is the focus of this PhD research, appears to be lagging behind. These days, the design of building components is more about meeting technical requirements rather than finding optimal solutions. According to Alexandrov [15], "this translates to an emphasis on series of disciplinary simulations and individual disciplinary optimizations, with results reconciled among disciplines". A product is designed by a multi-disciplinary team where each member knows his/her field. This means that a great amount of time is spent managing the design information and performing many costly design iterations. Hence, the success of the design solution heavily relies on the experience and communication qualities of the team. Furthermore, the growing complexity and size of modern engineering systems makes the use of traditional design methods increasingly challenging.

Likewise, the adaptation of new technology in the construction industry is slow compared to other industries when it comes to 3D printing, or 'automation in construction' in general. Nevertheless, the concept of 'Construction 4.0' gradually begins to impose itself [16,17]. According to [18], the construction sector has three primary opportunities for increasing productivity. The first is an increased automation of traditional physical tasks on site. For example, robot-laying bricks and robotic assisted timber construction [19,20]. The second opportunity comes from the automation of production in factories, where 3D concrete printing plays an important role (see Chapter 5) [21,22]. A third opportunity centers around digitalization in general, influencing all aspects of the design phase, planning, and management procedures [23-25]. With respect to the latter, building information modeling (BIM) is an essential technological development that has shown great potential by providing systems that ease the way of handling data and, as such, boosting collaboration projects.

Other significant advances in the field of digitalization in design are made on concepts like parametric modeling and computational design. Parametric modeling is basically an enhanced modeling process with the ability to change the shape of model geometry whenever this would be required, without needing manual adjustments. On the other hand, computational design stands for the merging between traditional design methods and

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computational technologies. Computational design strives towards a more generative process where the output is no longer created by humans using drawing tools, but rather by auto-generated sets of instructions, variables, and parameters. In this regard, topology optimization can be seen as a subset of computational design.

While these technologies may not be considered as disruptive as for example 3DP in the automotive industry, they are gaining widespread adoption [26]. For example, parametric modeling tools are currently implemented in most major BIM software packages (e.g. Tekla, Revit, ArchiCAD, Allplan) and as mentioned before, many commercial CAE software packages include some form of TO already. But it is not because the tools are available, that they are being used. According to Ramanauskas [27]: "Most geometry in BIM models is still modelled fully manual, without wasting too much effort in defining parametric relationships (unless the software automatically manages to take some of that away from the user's manual effort)". And as stated by Saunders [28]: "Note that the quality of TO depends on the quality of the defined initial conditions – if assumptions or simplifications are made, then the analysis may be flawed." However, these new tools (and mainly skills) will surely grow over time and their influence will further increase. Maybe they will not completely replace the existing classic design process, but they might very well enhance and enrich it.

Additionally, the increased acceptance of computational design in construction projects can be linked to the rise of certain new production methods. Many of these innovative production methods offer new design freedom and open up unexplored architectural possibilities. 3D concrete printing (3DCP) is one of such recent (but challenging) production techniques. It uses robotic-controlled concrete extrusion processes in order to fabricate functional building and construction components. In contrast to traditional manufacturing, 3D objects are produced by stacking layers of concrete (or cementitious material) on top of each other in a layer-wise fashion [29]. Like all AM techniques, 3DCP is expected to enable the production of shape-complex and mass-customizable objects where the cost of production should not increase with complexity [22]. A nice bonus is

that it could potentially also reduce CO₂ emissions and energy consumption by a large factor compared to a precast counterpart [30].

1.2 Problem definition

First of all, when designing products for the construction industry, a typical design process is influenced by many factors. These can be cost, environmental impact, available materials, but also multi-physics aspects like heat transfer optimization, acoustics, and fire safety. Designing construction-related components is unavoidably a very "multi-disciplinary" activity. Particularly, when considering the building envelope, being the physical separator between the interior and exterior of a building, many different aspects need to be considered. Finding ideal solutions for all design problems may require knowledge of both structural mechanics, thermal analysis, hygrothermal effects, durability, sustainability, fire safety, etc. Adding to this, the increasing demand for energy neutral housing and the growing technical requirements for these elements, a multi-physics approach can certainly be of great importance to the industry. As no well-defined framework or design strategy for multi-physics optimization is currently available, the aim of this doctoral research is to respond to this and aid the construction industry in the optimization of their products in a more efficient manner. The first scientific objective that tries to solve this problem is described in Section 1.3. Part A.

A second problem that is addressed in this PhD thesis is the link between topology optimized design and existing production techniques for the construction industry. TO's current ability for creating optimized, sustainable, and weight-efficient structures is largely underexploited. Firstly, it is very hard to produce these optimized structures with current and conventional manufacturing techniques. And secondly, because of the novelty of 3D concrete printing techniques, the process-specific limitations and constraints imposed by the manufacturing process are still mainly unclear. Designs that are optimized according to specific TO algorithms (in their current implementation) may behave unexpectedly or even suffer from damage because of the unforeseen consequences of the digital

fabrication process. A second scientific objective is defined to solve this aspect and is described in Section 1.3.1. Part B.

1.3 Objectives

The objective of this PhD project is twofold. Firstly, the aim is to **improve the design** process of construction-related components using a multi-disciplinary approach and take into consideration the most efficient production method. The ambition is to demonstrate the efficiency and effectiveness of newly developed optimization methods for problems that are complex and non-intuitive. Current optimization techniques will have to be analyzed, evaluated, and improved upon. Secondly, **the synergy between highly optimized designs and additive manufacturing techniques for the fabrication of these complex shapes is investigated**. While 3D concrete printing and topology optimized designs for the construction industry go hand in hand, the current challenges and opportunities for structural engineering of digitally fabricated concrete components are explored.

Part A - Multi-physics topology optimization of building components

First of all, there is the need to acquire critical reasoning with respect to the formulation of the complex design problems. This means investigating how to decompose a realworld problem into its disciplinary models and reintegrate them into a general (perhaps simplified) optimization formulation. The appropriate objective functions, constraints and derivatives need to be defined, taken into account, the concept of 'value-driven design'. This will require critical evaluation of multiple case studies and techniques to quantitatively assess the impact of each design criterion.

This part is aimed at improving existing shape and topology optimization methods for building (envelope) components and taking into consideration some multi-disciplinary performance indicators (e.g. structural and thermal performances). The goal is to strengthen the global search for lighter and more sustainable building envelope design.

Part B – 3D-printed topology optimized concrete structures

A second aspect of this PhD study focuses on bringing together two (e)merging technologies that show great potential for realizing structurally efficient building components: (i) Topology optimization for simulation-driven design; and (ii) 3D printing for the manufacturing of the resulting complex optimized shapes.

However, as to date, most examples made by 3DCP experiments have been rectilinear, solid, and their design based on long-established and familiar shapes. It is clear that current 3DCP experiments focus on improving the manufacturing process, and not yet on the design. More often, design complexity is enforced by designers and architects for aesthetic reasons only, while the structural design opportunities envisioned by digital fabrication are somewhat being neglected. This is mainly attributed to the fact that appropriate tools to fully exploit them, also in terms of a commonly agreed 'tailored' design and code-regulation framework, may not yet be fully available.

Automated design methods such as TO and other generative design techniques have been around for quite some time and can generate optimized designs in many applications. TO processes are very efficient on computational resources and have proven to deliver trustworthy results.

1.4 Valorization

The economical vision behind this research study is that it should trigger a "chain of improvements" in the construction industry. This is achieved by enhancing the design process of building components and materials, creating benefits, and stronger competitive positions to successive players in the market. The main potential or added value can be found with the manufacturer companies. Enhancing the design process means that the path to the final, optimized product is shorter. This implies spending less time developing the product (time is money), in cheaper ways, and consuming less raw material and energy. All of this is giving an advantage to the engineering design team.
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Once an improved building component (such as a thermally optimized masonry block or a thermally efficient brickwork support bracket) is created, and it seems fit for application, it is to be expected that some of them can be produced in a more economical way: using less energy and consuming less material. For other building components, it may be more expensive or complex to produce them, but the final applicants may have other economic interests such as better labor efficiency in complex constructions. Logically, an engineering team would only design products with economic advantages.

Secondly, the development of optimized construction methods and materials will also have a favorable effect on the actors in the building sector that apply these systems. Contractors will be able to purchase a better product, at a lower price, and/or needing fewer efforts to comply with the technical constraints of their projects. This delivers a benefit on several fields, like the use of energy, materials, labor, and machinery.

It is expected that if one development is successful, this success will further stimulate the market to further create more building components and materials that have improved qualities. A kind of dynamism will be created. In consequence, the market for successful new products will interest entrepreneurs in the building industry, and be applicable, on a larger scale. This is what is meant by "a chain of improvements".

Imagine a building component that is designed by such multi-disciplinary approach: complying with all the technical requirements, producible with X% less energy, and using Y% less raw material. Apply to this, the effect of scale (due to its success), even if X and Y are small, the profitability of the company, and its competitive position can be substantial.

The construction sector contributes to a significant part of the Gross National Product in Flanders and Belgium, by consequence, any small improvements will have a large effect. This novel approach will be beneficial to the whole sector and will increase its competitiveness in the international market. If the market players at these different levels would each of them strengthen their market position and become more competitive, this would finally lead to a boost in the whole economy.

In summary: using relatively few resources, a limited improvement in design (found at the beginning of the whole process) will lead to a chain of stronger positions for companies at different levels. This creates a leverage with benefits to a substantial part of the total economy, thus having a huge multiplier effect.

1.5 Methodology and thesis outline

This doctoral thesis contains in total 6 chapters (see also Figure 1.1). Each chapter will discuss one or more specific problems, chosen specifically to answer one of the research objectives from the previous section.

The overall process of the flow of the research that is conducted mainly follows that of a case study approach. The advantage of using case studies is that it allows for the exploration and understanding of different aspects of a complex topic, without losing focus. Each case study can provide new and unexplored insights into a subject, while some will even open up new research directions that steer the PhD study in a certain direction. A disadvantage of the case study research approach is that it can potentially provide little basis for generalization of results towards wider application. Also, the researcher's own subjective feeling may influence the setup of the case study or choice of certain study parameters (researcher bias). A complete discussion on these aspects can be found in [31]. However, most disadvantages of the case study approach are developed with respect to social science research studies, where experiments are typically conducted by psychologist and performed to test specific hypotheses. Nevertheless, in the field of engineering, case studies have been particularly important in the generation of new ideas and theories. Furthermore, [32] states that "the case study approach has been proven reliable to capture the rich information for the purpose of the investigation". Therefore, it is suggested as an important research strategy and method.

To ensure that the problem statement of each case study is relevant and has solid academic grounding, each chapter starts with presenting the state-of-the-art knowledge and literature, whereafter, the theoretical framework is being developed. This theoretical

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framework identifies the key concepts and theories that will be applied further on. This is described as the method or methodology of the chapter. In this work, extra attention is given to make sure the results are interpreted as objectively as possible. In addition, all necessary data is provided to make each case study reproducible, if this would be needed. Each chapter is always concluded by an exhaustive discussion and reflection of the findings. Taking these aspects into consideration, the methodology as used throughout this thesis should also allow for some preliminary generalization towards wider application and create a stronger basis for future research.

The first and current chapter (Chapter 1) discusses the general context and scope of this PhD study. The broad context of topology optimization in relation to the construction industry is elaborated, along with a problem definition, objectives, and outline of this research study.

Chapter 2 presents the relevant state-of-the-art of design optimization for the construction sector and discusses several distinct optimization methods and algorithms. The focus lies on density-based topology optimization, and an overview of some optimization strategies and examples is given.

Chapter 3 gives the theoretical framework that is required for simultaneous structural and thermal topology optimization, and a multi-material topology optimization approach is developed. Both methodologies are explored, and their effectiveness is confirmed using case studies. Firstly, the optimization of a thermally efficient masonry block is presented. Secondly, an optimized design for an efficient brickwork support bracket is demonstrated.

In Chapter 4, the link between topology optimization and additive manufacturing in the construction sector is discussed. These two (e)merging technologies truly stimulate one another and allow for the creation of new penalization schemes in topology optimization that are especially useful for the design of 3D-printable building components.

Chapter 5 proudly presents a unique realization in the field of digital fabrication for the construction industry: a topologically optimized post-tensioned concrete girder designed

and produced during this PhD study. An overview of the complete design and production process is elaborated.

Finally, Chapter 6 provides conclusions on the complete work and recommendations towards future related studies. It ends with a futuristic outlook on the (far) future of combined design optimization and digital fabrication.



Figure 1.1. Schematic that illustrates the structure of this PhD dissertation.

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CHAPTER II

OPTIMIZATION METHODS & ALGORITHMS

2.1 Mathematical optimization

Mathematical optimization is a process where the best possible value of some objective function is sought for by adjusting and finding the best input for the variables of the (design) problem. A general framework can be represented in the following way:

Find
$$\mathbf{x} = \begin{cases} x_1 \\ x_2 \\ \dots \\ x_n \end{cases}$$
 for which $f(\mathbf{x})$ has the lowest value

$$\vdots \\ x_n \end{cases}$$
(2.1)
with
$$\begin{cases} g_j(\mathbf{x}) \le 0, \quad i = 1, \dots, m \\ h_j(\mathbf{x}) = 0, \quad j = 1, \dots, n \end{cases}$$

Where, **x** is the vector containing the design variables and $f(\mathbf{x})$ is the objective function that is dependent on all design variables. Additionally, in this problem formulation, two sets of constraint functions, $g(\mathbf{x})$ and $h(\mathbf{x})$ can be defined which set conditions for the variables that are required to be satisfied. Although, this general framework may seem rather simplistic, many real-world and theoretical problems may be modeled with it. The key remaining question is: how the best values for these design variables **x** can be found?

2.1.1 Evolution towards gradient-based optimization methods

A huge diversity of different optimization methods and strategies exists [1,2]. The type of optimization method (the solver) greatly depends on the problem that needs to be solved and its efficiency can be measured as the time that is required to solve it. For many strategies, a trade-off exists between solution quality and effort; the higher the effort, the higher the solution quality can be.

The simplest type of optimization solver must be the brute-force search algorithm [3]. It is a general problem-solving technique that consists of systematically enumerating all possible candidates for the solution and checking whether each candidate satisfies the

problem's statement(s). The main disadvantage of the brute-force technique is that for most real-world problems, the number of candidates is prohibitively large.

Many alternatives to brute-force search have been explored [4]. From making early cutoffs of parts of the search space, to treating the design vector as living creatures that move around according to biological rules. In recent years, the latter, mostly, non-gradient-based methods such as simulated annealing, particle swarm optimization and other genetic algorithms have become very popular. The reason for this is the many advancements that were made in computer-based modeling and the increased efficiency of modern analysis methods, allowing engineers and designers to rapidly simulate the performances of a design in a virtual environment. Using these so-called "heuristic" methods, the swift generation and evaluation of many design alternatives has been given good results.

However, because of the complexity and size of many real-world engineering problems, the use of these methods can still be too computationally expensive (expressed in computation time and memory). Parallel computing can aid in overcoming this obstacle, but it cannot counter for computational inefficiency [5].

Therefore, gradient-based optimization methods offer a respected alternative. With the use of direct computational optimization techniques and differentiable functions, design problems can be optimized using function derivatives and gradient information (sensitivity information). This means it uses the gradient of the functions to determine the most promising directions along which we should search for the best values of these design variables [6]. For problems with many variables, like topology optimization, gradient-based methods are usually the most efficient. While non-gradient methods for topology optimization do exist, most of them were heavily criticized by many. In literature, a critical review by Sigmund [7] can be found that elaborates on this aspect.

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2.1.2 MMA and GCMMA

Structural optimization problems are often solved using a first-order method, where an initial design is updated in an iterative manner, based on gradients. The gradient-based optimization solvers that have been used almost exclusively in this work are two methods developed by Svanberg [8]: the method of moving asymptotes (MMA) and its successor, the globally convergent version of MMA (GCMMA). The methods represent a family of convex approximation methods suitable for structural optimization problems with many (10k and more) design variables. The standard form for nonlinear programming with MMA is represented in the following way:

minimize $f_0(\mathbf{x})$

subject to
$$\begin{cases} f_i(\mathbf{x}) \le 0, & i = 1, 2, ..., m \\ \mathbf{x} \in X \end{cases}$$
 (2.2)

where f_0, f_1, \ldots, f_m are given differentiable functions and $X = \{\mathbf{x} \in \mathbb{R}^n \mid x_j^{\min} \le x_j \le x_j^{\max}, j = 1, \ldots, n\}$ where x_j^{\min} and x_j^{\max} are given real numbers which satisfy $x_j^{\min} < x_j^{\max}$ for all *j*. As described in [8] and with respect to the ordinary MMA approach: "In each iteration, the current iteration point $\mathbf{x}^{(k)}$ is given. Then an approximating subproblem, in which the functions $f_i(\mathbf{x})$ are replaced by certain convex functions $\tilde{f}_i^{(k)}(\mathbf{x})$, is generated. The choice of these approximating functions is based mainly on gradient information at the current iteration point, but also on some parameters $u_j^{(k)}$ and $l_j^{(k)}$ (upper and lower asymptotes, the "moving asymptotes") which are updated in each iteration based on information from previous iteration points. The subproblem is solved, and the unique optimal solution becomes the next iteration point $\mathbf{x}^{(k+1)}$. Then a new subproblem is generated, etc."

2.2 Structural optimization methods

Within the field of structural optimization, a distinction can be made between three groups of general optimization strategies: sizing, shape and topology optimization [6]. Sizing optimization is the simplest and best-known form of structural optimization. Here, the general design of the structure is already decided on, and the aim is to adjust the dimensions of the various structural components as such that an optimized result (most material-efficient or cost-effective structure) is obtained. In this case, the design variables are the dimensions of the elements (e.g., type of the HEA profile or the thickness of the beam's flange) [9] (Figure 2.1a).

In shape optimization, the general design is also available at the starting point of the optimization. However, now the control points of the structural components themselves can be changed by the optimization algorithm. For example, the start and end positions of the structural members can be moved in space, in an attempt to improve the system's structural performance (e.g. an improved structural stiffness). Another specific application of shape optimization works by adjusting the control points of non-uniform rational B-splines (NURBS) that may represent the boundaries of a structural shape (Figure 2.1b). By shifting these points, an optimized shape can be found [10], or stress concentrations at certain locations can be removed [11]. Like sizing optimization, shape optimization will not alter the structural design (i.e. the topology of a design) significantly, since it cannot introduce new members or add holes to the structure. The boundaries and general layout were fixed beforehand.

Topology optimization, on the other hand, solves the fundamental problem of distributing a limited amount of material in a predefined design space [12]. With this method, no prior knowledge of the structural lay-out is needed. Only the problem itself (objective and constraints functions) needs to be well-formulated. The method enables the exploration of new design concepts and is most useful at an early stage in a design process (Figure 2.1c).



Figure 2.1. Three groups of optimization methods, (a) sizing, (b) shape, and (c) topology optimization.

Over the past three decades, different types of topology optimization have been studied: so-called continuum and ground-structure optimization. The term ground-structure optimization refers to the discrete formulation of the design space. The structural domain is discretized in a finite number of spatial nodes and a lattice-like structure is created. After the optimization process, only the most essential truss members are retained with respect to the prescribed loads and performance criteria (Figure 2.2a). The origin of this type of optimization can be traced back to 1904 [13] when Michell derived the formulas for structures with minimal weight under a given stress. These structures were called Michell trusses and have maximum rigidity for the available volume. They were considered global optimum. The disadvantage of this method is that the computation time increases as the number of nodes increases. Because of this, its application is somewhat limited to the simpler problems. Nevertheless, advances in the field [14] revealed efficient approaches for solving truss layouts in large-scale applications, because the truss layout is a linear problem that can be solved efficiently using interior point methods, that scale very well. However, these methods fall outside the scope of this thesis.

In continuum topology optimizations, the design domain is modeled as continuous spatial media whose motion and equilibrium are governed by balance laws and constitutive relations. The problem is most often solved by discretizing the design domain into finite elements and the material densities (micro-structure) of these elements are then treated as the problem variables (Figure 2.2b). Continuum topology optimization was first

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introduced by Bendsøe and Sigmund [15] and has become a major tool for conceptual structural optimization. Applications are not limited to the field of structural mechanics, but also lend themselves for multi-physics optimization problems like heat transfer, dynamic, acoustic, electromagnetic, multi material, and fluid flow problems [16].



Figure 2.2. Two types of topology optimization: (a) ground-structure (discrete) optimization and (b) continuum topology optimization.

2.3 Continuum topology optimization

Within the field of continuum topology optimization, most often the density of each element in the finite-element discretized domain is used to construct the vector of variables. This element density is often referred to as x_e , (or ρ_e) and is generally continuously variable between 0 and 1. As such, $x_e \in [0,1]$. A value of 0 means the material is absent (void), and a value of 1 means the material at this location is fully present (solid). Furthermore, each variable influences the physical material properties that are associated with the element. E.g. for mechanical problems this could mean that the value of x_e influences the Young's modulus E_e of the element, and for heat transfer problems, x_e could relay to the thermal conductivity λ_e of the element. For correct interpretation of intermediate values of x_e , interpolations schemes are needed and developed [17].

The SIMP method (or Solid Isotropic Microstructures with Penalization) is the most wellknown method for this to date. The method was proposed by Bendsøe in 1989 [18] and uses a simple power law function. Then, the Young's modulus of an element could be defined as $E_e = x_e^p E^0$, where p is the penalization factor, usually set to 3 and E^0 is the base material's Young's modulus (stiffness). The SIMP method states that under fairly simple restrictions on p, any stiffness used in the SIMP model can be realized as the stiffness of a small microstructure made of voids and an amount of base material. Additionally, the SIMP model is isotropic with a Young's modulus varying with x_e and a constant Poisson ratio, independent of x_e (Figure 2.3). Other penalization models that are sometimes used are the RAMP model (for dynamic situations) and the SINH model. For more information on this aspect, see Chapter 4 of this PhD study.

Solving the topology optimization problem is equivalent to finding the optimal values for the vector of design variables. In most cases, the use of proper interpolation schemes will



Figure 2.3. (a) SIMP interpolation curves for varying values of p, illustrating the relationship between relative stiffness (E_e/E^0) versus the density of an element (x_e), and (b) Microstructures of material and void realizing the material properties of the SIMP model with p = 3 and a base material with Poisson's ratio v = 1/3 (adopted from [17]).

make sure that the final material distribution is a black ($x_e = 1$) and white ($x_e = 0$) rasterized shape with sharp boundaries. As discussed previously, one way of finding the optimal value for the density vector would be to examine all possible combinations and analyzing them for efficiency. When only considering black and white elements (these are also called ISE elements (Isotropic Solid or Empty elements), the number of combinations that can be made is 2^N , where *N* is the number of elements that make up the discretized design domain. Taking into account that a typical finite element mesh consists of more than 10.000 elements, it goes without saying that this method is not very practicable. As such, for topology optimization problems, gradient information is used almost exclusively to find an optimized material distribution in an efficient manner. Also, a schematic of all steps required in such process is shown in Figure 2.4. In Section 2.4 a general overview of a classic topology optimization implementation is presented.



Figure 2.4. General scheme of a classic topology optimization process using the SIMP method (using sensitivity filtering).

2.4 Compliance minimization

2.4.1 Problem formulation

The most 'simple' form of topology optimization can be formulated as a procedure where the influence of the external loads and the associated deformations on the structure, (also defined as the compliance of a structure), is to be minimized. This is equivalent to maximizing the stiffness. Of course, the stiffest design is a structure where the complete design domain is filled with material. Therefore, a volume constraint is imposed in addition to this objective function. In literature, this is also called 'Compliance minimization with a volume constraint'. Mathematically, the compliance of a structure is equivalent to the total elastic strain energy, and the complete problem formulation can be written as:

minimize compliance:
$$C(\mathbf{x}) = \mathbf{F}^{T} \mathbf{U}$$

subject to: $V(\mathbf{x}) / V_{max} - 1 \le 0$
 $\mathbf{K} \mathbf{U} = \mathbf{F}$
 $0 \le \mathbf{x} \le 1$ (2.3)

where $C(\mathbf{x})$ is the compliance, \mathbf{F} is the global load vector, and \mathbf{U} is the vector containing the elastic displacements. $\mathbf{KU} = \mathbf{F}$ represents the balance equation for mechanical systems where \mathbf{K} is the global stiffness matrix. \mathbf{x} is of course the vector containing all design variables x_e (the density of each element), and $V(\mathbf{x})$ and V_{max} are respectively the material volume of the structure and the maximum allowable volume of the system.

2.4.2 Modified SIMP interpolation

In Section 2.3, it was mentioned that a power law function can be used to map the element densities to the Young's modulus and that this can be written as: $E_e = x_e^{\ p} E^0$. However,

in order to prevent the stiffness matrix from becoming singular when $x_e = 0$, a small modification should be made. This modified SIMP formulation [20] reads as follows:

$$E_e(x_e) = E_{\text{void}} + x_e^p(E_{\text{solid}} - E_{\text{void}})$$
(2.4)

where E_{solid} is the stiffness of the base material and E_{void} is a very small value attributed to eliminate the predefined singularity problem.

2.4.3 Sensitivity analysis

In order to formulate the MMA subproblem and create the approximating functions $\tilde{f}_i^{(k)}(\mathbf{x})$ as described in Section 2.1.1, the partial derivatives of all used functions (objective and constraint functions) with respect to the design variables x_e are needed. This process is also called the sensitivity analysis of our system and is performed using either approximate, numerical methods, or exact analytical methods.

As described by Holmberg [19]: "this traditional minimum compliance formulation has gained its popularity much because **K** depends linearly on **x**." In addition, " $C(\mathbf{x})$ is a so-called self-adjoint function, which makes it computationally very efficient because no additional linear system needs to be calculated to obtain the gradients." As such, for cases where the loads are design-independent, the sensitivities of the objective function $C(\mathbf{x})$ are found using the adjoint method and the derived equation is presented in Eq. (2.5). Also, the sensitivities of the material volume $V(\mathbf{x})$ (for finite elements with a volume of 1) are given.

$$\frac{\partial C(\mathbf{x})}{\partial x_e} = -\mathbf{U}(x) \frac{\partial \mathbf{K}(\mathbf{x})}{\partial x_e} \mathbf{U}(\mathbf{x})$$

$$\frac{\partial V(\mathbf{x})}{\partial x_e} = 1$$
(2.5)

See Appendix 1, for the calculus behind this derivation. A complete review of adjoint methods for sensitivity analysis in numerical codes can be found in [21].

2.4.4 Filtering

The use of filters has proven to be beneficial during topology optimization for several reasons. One possible problem is the appearance of alternating solid and void elements (checkerboard patterns) which can create structures that have an artificially high stiffness (Figure 2.5). Alternatively, filters can also be used to impose a minimum length-scale control (constraining the minimum feature size).



Figure 2.5. The appearance of the checkerboard pattern in TO problems when no sensitivity or density filter is used.

Throughout this PhD study, most often a density filter [22] is used in order to avoid the formation of checkerboard patterns. The original densities are transformed as follows:

$$\tilde{x}_e = \frac{1}{\sum_{i \in N_e} H_{ei}} \sum_{i \in N_e} H_{ei} x_i$$
(2.6)

In this equation, N_e is the set of elements *i* for which the center-to-center distance (*e*, *i*) to element *e* is smaller than the filter radius r_{\min} , and H_{ei} is a weight factor defined as:

$$H_{ei} = \max\left(0, r_{\min} - \Delta(e, i)\right) \tag{2.7}$$

When applying a density filter, the sensitivities of the objective function and the constraint functions have to be adjusted. The determined sensitivities are still relevant. However, the variable x_e is replaced with \tilde{x}_e . Additionally, the sensitivities with respect to the design variables x_k are obtained by means of the chain rule [22]:

$$\frac{\partial \psi}{dx_k} = \sum_{e \in N_k} \frac{\partial \psi}{d\tilde{x}_e} \frac{\partial \tilde{x}_e}{dx_k} = \sum_{e \in N_k} \frac{1}{\sum_{i \in N_e} H_{ei}} H_{ke} \frac{\partial \psi}{d\tilde{x}_e}$$
(2.8)

where the function ψ can represent either the original objective function or constraint functions.

2.4.5 Example

In the example below, the well-known MBB problem [23] and its optimized shape are presented (Figure 2.6) to illustrate the functionality of the TO algorithm that was elaborated in Section 2.4.1. The MBB problem is a classic benchmark problem for TO, consisting of a simply supported beam, loaded with a vertical force centered on its top boundary. The problem originates from a real-world design challenge disclosed by a West German aerospace manufacturer, Messerschmitt-Bolkow-Blohm, hence the name MBB. A more detailed description of this problem is given in a textbook by Bendsøe and Sigmund [12]. In this study it is only used as a first demonstration of the capabilities of TO.



Figure 2.6. (a) MBB-beam design problem and (b) its topology optimized design, having a material reduction of 65% with respect to the solid design domain.

2.5 Extensions

Compliance-based topology optimization is the most iconic and well-established form of TO. However, in literature many other and more advanced implementations have been developed. In this section, a few alternatives are briefly presented to give a general overview of the capabilities of TO for construction related applications. For more detailed information on these alternative implementations, the reader is referred to the referenced publications.

The first extension provides a method to optimize the natural frequencies of a design (also see [24,25], the second approach takes into account von Mises stresses (also see [26,27], and the third extension elaborates on the differences between TO using small deformation theory and large deformation theory (also see [28]). With respect to the latter, also the influence of topology optimization with variable loads and/or multiple load combinations and -scenarios is reviewed and linked to different approaches to perform a multi-objective TO. Finally, a preview of topology optimization in 3D is also presented (also see [29]).

2.5.1 Natural frequency TO

Free vibration occurs when a mechanical system is set in motion with an initial input and can vibrate freely. A mechanical system will then vibrate according to one or more of its natural frequencies. In addition to compliance-based TO, the optimization of natural frequencies and free vibrations has been studied by many. Its purpose is to ensure that the natural frequencies will not be similar to the occurring and forced frequencies of the structure. In most cases, we would want the fundamental frequencies to be far off so that no resonant frequency could potentially harm the structure.

Some studies have also shown that structures with high fundamental frequencies are relatively stiff for many different load cases. Thus, topology optimization with the aim of maximizing the fundamental frequencies has been giving good results for static design problems as well. Figure 2.7 presents two examples for which the first natural frequency was maximized.



Figure 2.7. Topology optimization where the 1st natural frequency is being maximized. Figures (a) and (c) present the design problems and boundary conditions, and (b) and (d) show the optimized structures respectively.

2.5.2 Stress-based TO

Another important implementation is stress optimization in TO. Stress optimization is one of the key factors for structural design in construction, especially when fatigue plays a vital role in the design. As mentioned before, when a design is optimized towards minimum compliance, this does not necessarily mean that a good stress distribution is obtained. In addition, the maximum allowable stress may have been exceeded in some locations. Especially, when a design has a lot of sharp boundaries or predefined regions of voids, a different implementation should be considered. In such situations, the topology optimization problem is better optimized with stress constraints or a formulation that takes stress as a parameter in the objective. Figure 2.8 illustrates the difference between a compliance-based optimization and a stress-based optimization in case of a L-bracket design. In this example, the design domain is characterized by a right angle, which causes peak stress concentrations at this discontinuity. This design problem is the classic benchmark study for this type of problems [30]. In the result where the von Mises stresses were minimized (Figure 2.8b), it can be seen that the peak stress is lower due the design

having a rounded corner at the discontinuity instead of a right angle. However, the deflection is somewhat larger.



Figure 2.8. The difference between (a) compliance-based and (b) stress-based optimization for the L-bracket benchmark study. Figures (c) and (d) show the relative differences in deformations and peak stress concentrations in the right-angled corner for equal loads and identical material reduction.

2.5.3 TO based on large deformation theory

So far, all TO algorithms assumed small deformation theory. This means that the deformation of the structure under its loads is neglected. In normal situations this does not form any problem, as these optimized designs general perform well for any magnitude of the load. However, in a small number of design problems, it might be beneficial to include large deformation theory. In large deformation theory the deformation caused by

the load is taken into account, and a geometrically nonlinear finite element analysis should be included in the optimization formulation. Figure 2.9a and b demonstrate the differences between TO with small and large deformation theory. In the latter, a different design was found that takes into account the deformation of the structure under its load.

At first sight, this optimized solution does not look very optimal, as the structural element at the beam's end is subjected to bending stress. However, when looking at the actual deformation of this design under its design load, as shown in Figure 2.9c, it immediately becomes clear. The structural element is not subjected to any bending stress, but rather is in pure tension. An important aspect here is that the resulting topology is only optimized for the considered design load (direction, size, type, position, etc...). For example, a small change in the magnitude or direction of this load could create a weak link in this structure. As such, the resulting topology is only optimized for the considered design load. A general overview on how to use topology optimization in the light of variable loads and/or multiple load combinations and scenarios is presented in the next section.



Figure 2.9. Difference between TO that assumes (a) small deformation theory and (b) large deformation theory. (c) presents the deformed state of the optimized structure that considers large deformation theory.

2.5.4 TO with multiple load conditions

As explained in [12], the framework described for minimum compliance design for a single load case can be generalized quite easily to a situation where it is influenced by multiple load conditions. I.e., the original framework is transformed from the minimization of a single compliance function to the minimization of a weighted average of the compliances for each of the load cases. Beforehand, a complete list of all possible load combinations and scenarios is defined, and these are then combined into the list of load cases. The main solution strategy includes the weighted-sum method. In such formulation, the (in theory) multi-objective optimization approach (optimizing for all load cases separately) is converted into a single-objective optimization function (also referred to as the 'weighted-sum multi-objective' in this work). Similarly, when using the MMA algorithm, the sensitivity of the weighted-average function of the compliances becomes the weighted average of the sensitivities of each of the individual compliance functions. Finally, it can also be remarked that the inclusion of extra load cases is computationally cheap since the stiffness matrix can be factorized [12].

2.5.5 Multi-objective TO

The weighted-sum multi-objective optimization approach is one of the most widely used methods in multi-objective TO mainly due to its simplicity. Nonetheless, the work of [31] states that it can be difficult to generate a good set of points that are uniformly distributed on the Pareto front using such formulation. The problem is that the linear weighted sum method only works for problems with convex Pareto fronts. In the referenced work, a simple example is presented where two objective functions are combined into one for a given set of $w_1 + w_2 = 1$, and the composite function F is minimized (Figure 2.10). For any given set (w_1, w_2) , a (dashed) line has a gradient $(1, -w_1/w_2)$ that will become tangent to the Pareto front when moving downward to the left, and that touching point is the minimum of F. However, at the nonconvex segment, if the aim is point C, the weighted sum method will usually lead to either point A or point B, depending on the values of w_1 (since $w_2 = 1 - w_1$).



Figure 2.10 Weighted-sum method for two objectives f1 and f2 and w_1 and w_2 . Illustration adopted from [31].

Furthermore, proper scaling and normalization operations of the objective functions are required so that the ranges/values of each objective should be comparable. Otherwise, the weight coefficients are not well distributed and can thus lead to biased sampling on the Pareto front. Nevertheless, the limitation discussed above is not specific to TO, but was elaborated from a broad mathematical standpoint. Excellent examples of multi-objective TO under multiple loads can be found in the work of Hongwei et al. [32], Peng et al. [33], and Stanford and Ifju [34] who optimized respectively a hybrid electric vehicle frame, an aluminum-alloy-bus body component, and a flexible wing skeleton for micro and small unmanned air vehicles. In all studies, the authors reported no problems using the weighted-sum multi-objective approach and were able to produce valid Pareto-optimal results by modifying the weights assigned to the individual objectives.

In contrast, a recent review paper by Marler and Arora [35] discusses other methods to overcome the limitations of the weighted-sum multi-objective, derived from the need to assign a relative importance to each criterion. A posteriori articulation of preferences and a generate-first-choose-later approach are presented. The normalized 'Normal Constraint' method [36] belongs to this second group, and according to its authors it can produce evenly spaced Pareto frontiers [37], which are independent of the scale of the individual

objectives. Nevertheless, this recent review paper claims that thus far, there is little to no application of these methods in SIMP topology optimization.

Because a more detailed study of these alternative approaches was out of the scope of this work, the weighted-sum multi-objective approach was applied throughout this work.

2.5.6 Three-dimensional TO

Our world has three physical dimensions. However, all examples presented so far are flat, two-dimensional. One could argue that any real-world application would benefit from 3D optimization. However, while this is true, very often new TO algorithms are being developed in 2D only. Mainly because, when deemed functional, the extrapolation to the third dimension is not that complicated.

In order to demonstrate 3D topology optimization, the MBB design problem from Section 2.4.5 is extrapolated into the third dimension. Figure 2.11 presents the final distribution of material where the objective was to maximize the beam's stiffness with a constraint on the maximum amount of material used. The colors visualize the von Mises stresses. The result of Figure 2.11 was generated using the commercial software package Abaqus.



Figure 2.11. Three-dimensional topology optimization for the MBB design problem (colors showing the normalized von Mises stresses).

2.6 Reliability of results

2.6.1 Non-uniqueness, local minima

Many of the case studies investigated in this work make use of the SIMP method. In that respect it is important to know whether the results are reliable. SIMP is very often said to be largely heuristic and therefore it is only a question of luck that the correct solution is found. Nevertheless, the theoretical convergence of the SIMP method has been explored by many. For instance, when the SIMP method is reduced to the setting of the well-known variable thickness sheet design problem, the minimum compliance problem and the complete problem statement is a convex-concave saddle point problem that lends itself to a complete FE convergence analysis [38]. Also, Rozvany [39] said "SIMP is a reasonably rigorously derived gradient method for topology optimization, which usually gives a solution near the correct global optimum if the problem is originally convex (e.g. in compliance problems), and the penalty factor p is increased gradually from unity". Finally, SIMP has also been verified quantitatively by showing numerical convergence to Michell topologies for topology optimization [39].

Still, it is accurate to say that many real-world engineering problems in topology optimization are non-convex. Therefore, such problems can always have multiple local optima and non-unique optimal solutions. An example of the latter is de design of a structure in uni-axial compression or tension. In such a case, a structure consisting of one tick bar is equally as good as a structure containing many thin bars when the overall area of both structures is the same.

In contrast, studies by Rietz [40], Martinez [41], and Stolpe & Svanberg [42], found and demonstrated that SIMP-based methods require relatively few iterations and therefore are very suitable for a combination of a wide range of design constraints, multiple load conditions, multi-physics problems, and extremely large (often 3D) systems.

To conclude, using SIMP, a global optimum can often not be guaranteed, but usually, it gives a solution near or close to the global optimum.

Additionally, a critical review on the usefulness of non-gradient approaches in topology optimization by Sigmund can be found in [43], and a comparison between two of the most used methods for numerical topology optimization, namely SIMP and ESO is discussed in. The two Forum Discussion papers clearly promote the use of SIMP-based approaches, and even heavily criticize the other methods. For example, in the paper by Sigmund, it is argued that non-gradient approaches are hopelessly inefficient for problems with many variables such as topology optimization. It is also demonstrated that even for extremely coarse meshes a state-of-the-art non-gradient topology optimization does not even provide global optima.

2.6.2 TO in the construction industry

A second aspect that needs some discussion is the reliability of TO when used as a design tool in the construction industry. While in theory the optimized solution from a topology optimization study can be expected to improve the traditional design, very often certain assumptions are made to simplify the original design problem. As such, the response of the optimized shape or structure to the real-world boundary and load conditions might deviate substantially from the predicted mathematically calculated benefit.

Especially the application of TO in the construction industry might be subjected to many limitations. For example, the need of design for the Serviceability Limit State (SLS) and Ultimate Limit State (ULS) having in mind different load cases, different safety factors, and failure modes is very relevant, but not always considered in literature. In order to compensate for this, often post-numerical analyses are performed to verify the results and estimate the errors these simplifications introduced.

Additionally, the influence of different material properties can have a major influence on the optimized material distribution. For example, a design problem that will be made from concrete can and should look completely different when compared to a design made from steel. The importance of stress optimization for heterogeneous and non-isotropic

construction materials (such as concrete) should be a main point of attention but is often disregarded because of the increased computational cost and complexity of its implementation.

An excellent review paper on the state-of-the-art of TO for concrete construction can be found in [44]. The paper extracted over 200 research pieces from digital scientific literature databases and narrowed them down to 60+ relevant to the topic of TO in concrete construction. The paper outlines the relevant theoretical frameworks that currently exist and provides a general background on the different TO approaches with respect to concrete structural optimization. Furthermore, after extensive quantitative as well as qualitative review of the existing implementations and applications, the research gaps and a future vison on the topic are given.

As a supplementary note, also in this work many simplifications were made in the formulation of the numerous case studies. Special attention was given to clearly motivate the reasons for these assumptions and to discuss the impact of the respective limitations.

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CHAPTER III

MULTI-PHYSICS TOPOLOGY OPTIMIZATION

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1. G. Vantyghem, W. De Corte, V. Boel, and M. Steeman, "Structural and thermal performances of topological optimized masonry blocks," in Asian Congress of Structural and Multidisciplinary Optimization 2016, Nagasaki, Japan, 2016.

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2. G. Vantyghem, V. Boel, M. Steeman, and W. De Corte, "Multi-material topology optimization involving simultaneous structural and thermal analyses," Structural and Multidisciplinary Optimization, vol. 59, no. 3, pp. 731–743, 2019.

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3.1 Introduction

Up to this point in the PhD thesis, the topology optimization examples that were presented where mostly 'single physics' problems situated in the field of structural optimization. I.e. stiffness-based mechanical TO with a constraint on the volume fraction, and/or with stress constraints. Nevertheless, an increasing interested in multi-physics problems can be observed. Early examples of TO applied to multi-physics problems can be found in [1]. In recent years, topology design methods have also been expanded to for example electromagnetic problems, coupled problems, fluid problems, and wave propagation problems. [2-4]. Various challenges in applying topology optimization to multi-physics problems were defined by Sigmund in [5].

The term 'multi-physics', as used in this work, refers to using simulations that involve multiple physical models or multiple simultaneous physical phenomena. In this chapter, 'multi-physics' means: the simultaneous (but uncoupled) optimization of mechanical and heat transfer performances. This stands in contrast to some alternative definitions found in literature where the term is defined in a narrower sense as coupled or interacting physical studies.

This chapter focuses on improving existing topology optimization (TO) implementations in order to allow for the design of improved building envelope components. Also, a verification and several validations of the developed frameworks are presented and demonstrated by studying a number of case studies.

Because this chapter mainly deals with the optimization of problems in the conceptual design stage, some simplifications are made to reduce the complexity of the original problem formulation. E.g., in this chapter, the only mechanical performance indicator that is measured, is the stiffness (inverse of compliance). For the thermal performance, only conductive heat transfer is studied. Additionally, all analyses are performed in a steady-state condition. The motivation and legitimacy of these simplifications is given at the start of each of the case studies.

This chapter starts with an elaboration of the theoretical frameworks, followed by a first case study that presents the design of a thermally efficient masonry building block (Section 3.4.1). A second case study presents the optimization of an energy-efficient brickwork support bracket (Section 3.4.2).

3.2 Simultaneous structural and thermal topology optimization

This section presents a scheme for simultaneous structural and thermal topology optimization. While traditional structural TO problems optimize the stiffness of the design domain for a given fraction of material (as discussed in Section 2.4), in this chapter, the thermal performance of the design becomes the objective of the optimization formulation.

In the studies that will follow, the aim is to minimize the thermal transmittance (also Uvalue) through the design domain, with a constraint on the design's stiffness, and a limitation on the maximum material volume fraction that can be used. Therefore, an adjustment of the original problem formulation (minimum compliance) must be made. Analogous to minimizing the structural compliance, we can minimize the thermal compliance. This will result in finding an optimal thermal conductor. Conversely, by maximizing the thermal compliance, the thermal transmittance is minimized. Although the term compliance is not very often used in engineering in general, it has gained in popularity due to its numerical efficiency in topology optimization problems (i.e., due to its self-adjoint property, no additional FE calculation is required to obtain the gradients).

As TO with respect to steady-state heat transfer is very similar to that of static mechanical problems, an analogy can be made with the structural problem formulation. For the structural analysis, the state equation is given in Eq. (3.1) and the structural compliance (C_s) in formulated in Eq. (3.2):

$$\mathbf{K}_{\mathbf{s}}\mathbf{U} = \mathbf{F} \tag{3.1}$$

$$C_{\rm s} = \mathbf{U}^{\rm T} \mathbf{K}_{\rm s} \mathbf{U} \tag{3.2}$$

where \mathbf{K}_{s} is the global stiffness matrix, \mathbf{U} is the global displacement vector (mm), and \mathbf{F} the vector with the mechanical loads (newton). Very similar in form, the state equation for the thermal problem is defined as:

$$\mathbf{K}_{\mathsf{t}}\mathbf{T} = \mathbf{Q} \tag{3.3}$$

In this equation, \mathbf{K}_t is the global thermal conductivity matrix, \mathbf{T} is the vector containing the nodal temperatures (Kelvin), and \mathbf{Q} contains the thermal loads (Watt). Mathematically, the formulation of the finite element solver is identical, and the computation of the thermal compliance follows the same principle. The difference between the two analyses is that the thermal problem takes up less storage and will be faster to solve because it requires only one degree of freedom per node (θ_k), instead of two (u_k , v_k) or three in 3D. The compliance of the structure is a measure of work done by the load and is equivalent to the structure's internal energy (Joules). In the case of a linear elastic mechanical system, the compliance of a structure follows the strain energy equation, thus, $\frac{1}{2} \times$ stress × strain × volume = Nm (Newton-meter or Joule). For thermal problems, the basic requirement for heat transfer is the presence of a temperature difference. The units on the rate of heat transfer are Joule/second, also known as a Watt.

The mathematical formulation of such multi-physics approach reads as follows:

maximize:
$$C_{t}(\mathbf{x}) = \mathbf{T}^{T}\mathbf{K}_{t}\mathbf{T} = \sum_{e=1}^{N} \lambda_{e}(x_{e})\mathbf{\theta}_{e}^{T}\mathbf{k}_{t,e}^{0}\mathbf{\theta}_{e}$$

subject to: $C_{\rm s}(\mathbf{x})/C_{\rm smax} - 1 \le 0$

$$V(\mathbf{x})/V_{\max} - 1 \le 0 \tag{3.4}$$

with:

$$C_{s}(\mathbf{x}) = \mathbf{U}^{\mathrm{T}}\mathbf{K}_{s}\mathbf{U} = \sum_{e=1}^{N} E_{e}(x_{e})\mathbf{u}_{e}^{\mathrm{T}}\mathbf{k}_{s,e}^{0}\mathbf{u}_{e}$$

$$0 \le \mathbf{x} \le 1$$

where $C_{s}(\mathbf{x})$ and $C_{t}(\mathbf{x})$ represent the structural (s) and thermal (t) compliance respectively, and $V(\mathbf{x})$ is the material volume that linearly depends on \mathbf{x} . $C_{s \max}$ and V_{\max}

represent the maximum value of the corresponding functions C_s and V, and \mathbf{u}_e [8 (or 12) × 1] and $\boldsymbol{\theta}_e$ [4 × 1] are the element displacements and element temperatures vector. Likewise, $\mathbf{k}_{t,e}^0$ and $\mathbf{k}_{s,e}^0$ stand for the element stiffness and element conductivity matrix for an element (size: 1 × 1 × 1 mm) with unit Young's modulus and unit thermal conductivity, \mathbf{x} is the vector of design variables (i.e. the element densities), and N is the number of elements (e) used to discretize the design domain.

As discussed before, the variables of the design domain (x_e) are linked to the material properties of the elements. In the following studies, the modified SIMP method (Section 2.4.2) is used to map the variables to both the Young's modulus (E_e) in MPa and the element thermal conductivity (λ_e) in W/mK. This mapping of densities is performed using the following equations:

$$E_{e}(x_{e}) = E_{\text{void}} + x_{e}^{P_{s}}(E_{\text{solid}} - E_{\text{void}}) \quad x_{e} \in [0, 1]$$
(3.5)

$$\lambda_{e}(x_{e}) = \lambda_{\text{void}} + x_{e}^{p_{i}} \left(\lambda_{\text{solid}} - \lambda_{\text{void}}\right) \quad x_{e} \in [0, 1]$$
(3.6)

where E_{solid} is the Young's modulus of the solid material and E_{void} is the Young's modulus of the voids. Very similar, λ_{solid} and λ_{void} contain the values for material thermal conductivity. p_{s} is the penalization parameters of the structural interpolation function, and p_{t} is the thermal penalization parameter.

The sensitivities of the objective function C_s and C_t with respect to the element densities x_e are found using the adjoint method (Section 2.4.3 and Appendix 1) and presented in Eq. (3.7) and Eq. (3.8). The sensitivity of the material volume V is given in Eq. (3.9), where v_e is the element volume.

$$\frac{\partial C_{t}}{\partial x_{e}} = -p_{t} x_{e}^{(p_{t}-1)} (E_{\text{solid}} - E_{\min}) \boldsymbol{\theta}_{e}^{T} \mathbf{k}_{t,e}^{0} \boldsymbol{\theta}_{e}$$
(3.7)

$$\frac{\partial C_{s}}{\partial x_{e}} = -p_{s} x_{e}^{(p_{s}-1)} (\lambda_{\text{solid}} - \lambda_{\min}) \mathbf{u}_{e}^{T} \mathbf{k}_{s,e}^{0} \mathbf{u}_{e}$$
(3.8)

$$\frac{\partial V}{\partial x_e} = v_e \tag{3.9}$$

Additionally, a sensitivity filter was used. The filtering of sensitivities follows the methods described in Section 2.4.4 and the globally convergent version of MMA (GCMMA) is used to solve the minimization (and maximization) problem and update the design variables In addition, the maximum external move limit is set to 0.2, which is generally assumed to protect the over correction of design variables.

3.3 Multi-material topology optimization

In this section, a scheme for multi-material TO optimization is elaborated on. Previously, the finite element density was controlled by a variable with the following bounds: A minimum value of 0 and a maximum of 1. A value of 0 meant that the element was inactive or considered a void, while a density value of 1 meant that the element existed (solid). In order to implement a two-material approach, an additional element variable is added to the formulation. The goal is to allow the optimization algorithm to choose between two different types of solid material. The different materials are referred to as material 1 with material properties E_1 and λ_1 , and material 2 with material properties E_2 and λ_2 . In the following implementation, material 1 always has a lower thermal conductivity, meaning it is a good thermal insulator. However, this comes at the cost of its stiffness. Material 2 is the stiffer material but has a higher thermal conductivity (a good conductor of heat). In order to incorporate this two-material approach, Eq. (3.5) and Eq. (3.6) from the previous subsection are modified by replacing E_{solid} and λ_{solid} with the following:

$$E_{\text{solid}} = y_e E_2 + (1 - y_e) E_1 \tag{3.10}$$

$$\lambda_{\text{solid}} = y_e \lambda_2 + (1 - y_e) \lambda_1 \tag{3.11}$$

In these equations, $\mathbf{y} = \{y_1, y_2, ..., y_N\}^T$ is a newly added vector of design variables which behave similar as the first one $\mathbf{x} = \{x_1, x_2, ..., x_N\}^T$. While the original vector of

design variables \mathbf{x} is still in control of the presence of the voids, the second density variable \mathbf{y} is controlling the presence of one material over the other. The material properties of an element with an intermediate value (in y_e) are determined by the weighted average of material 1 and 2.

Of course, this adjustment influences the interpolation functions. As such, also a modification of the sensitivities is needed. Eq. (3.12) gives the sensitivities of C_s with respect to the variables x_e , while Eq. (3.13) gives the sensitivities with respect to y_e .

$$\frac{\partial C_{s}}{\partial x_{e}} = -px_{e}^{(p-1)}[(y_{e}E_{2} + (1 - y_{e}) E_{1}) - E_{0}]\mathbf{u}_{e}^{\mathrm{T}}\mathbf{k}_{s,e}^{0}\mathbf{u}_{e}$$
(3.12)

$$\frac{\partial C_{s}}{\partial y_{e}} = -x_{e}^{p} \left(E_{2} - E_{1} - E_{0}\right) \mathbf{u}_{e}^{\mathrm{T}} \mathbf{k}_{s,e}^{0} \mathbf{u}_{e}$$
(3.13)

A similar derivation can be made to calculate the partial derivatives of the thermal compliance C_t . Additionally, these equations should also be modified according to Eq. (2.7) to allow the filtering of the densities (as discussed in Section 2.4.4).

3.4 Case studies

3.4.1 Case 1: Thermally efficient masonry block

Traditional masonry blocks (or fired-clay bricks) were among the first artificial materials produced by men for building purposes [6]. They are considered easy to produce, resistant, and durable [7]. Most commonly, they are rectangular in shape and used to build in traditional masonry style with layers of staggered blocks, collectively known as brickwork. In Northern Europe, they became popular in the early Middle Ages because many places lacked indigenous sources of rock. The use of fired-clay bricks allowed these regions to develop specific styles of architecture, known as brick Gothic and Renaissance [8]. At that time, most bricks were solid and relatively small in size. Today, hollow bricks are used in many projects. Hollow clay bricks are vertically perforated bricks having the advantages being lighter and more efficient. They are used because they have better

strength characteristics and are economically more interesting. Additionally, they provide better thermal and acoustic insulation [9].

Insulating masonry building blocks exist in a variety of configurations. Some have multicores, others have interlocking parts. According to Lourenco and Vasconcelos [10], the design of an insulated perforated masonry block is defined based on three main parameters: (i) the structural behavior (associated to requirements of the construction system), (ii) its thermal performance, and (iii) ergonomics. The structural behavior is primarily defined by the compressive strength of the block, measured both perpendicular and parallel to the mortar bed. Other mechanical parameters such as shear, flexural strength, and robustness of the blocks exposed to combined vertical and horizontal inplane loadings are also important but can somewhat be ignored at a preliminary design stage. The thermal performances take the major role in the design of insulated masonry blocks. The geometry and arrangement of the internal cells heavily influence the thermal conductivity. Large perforations and insulated voids reduce the heat transfer through the block, but also make it weaker. Therefore, it is crucial to find the best possible material arrangement taking into account the minimal structural requirements. For fired-clay blocks, further geometric requirements can be found in Eurocode (EC) 6 which discusses the recommended percentage of holes, the thickness of the webs and shells [11]. Finally, concerning ergonomics, attention must be given to the ease of use of the masonry blocks, like i.e. limiting the maximum size of the blocks or having a center hole for easy manually handling.

The book 'Eco-efficient Masonry Bricks and Blocks' by F. Pacheco et al. [12] provides the starting point for this case study. In this work, an up-to-date state-of-the-art review about eco-efficiency of masonry units, their design, performance and durability is given. Pore forming techniques such as the insertion of organic material in fired clay bricks serve a possible solution to increase the thermal resistance. Cellular concrete is another widely used building component that has more or less the same effect. Another study in this book focused on the improvement of the general shape of blocks in order to improve their equivalent (and homogenized) thermal conductivity, clearly a TO problem.

In this case study, the thermal efficiency of a masonry block will be improved by finding an optimized cross-sectional material distribution, independent from material properties. This concept is also addressed by Bruggi and Taliercio in [13]. In order to optimize the cross-sectional area, topology optimization techniques are used. TO is considered the most suited method in contrast to traditional optimization methods that use parametric studies, as the optimized shape of an element is sought after based on clearly defined objectives, and boundary conditions, and without prejudice of the designer. In this case, the algorithm that was presented in Section 3.2 is adopted.

An essential aspect of the design problem is that the steel bracket is fully surrounded by insulation material. Therefore, thermal conduction is the dominant form of heat transfer and allows for the mathematical problem to disregard other, more complex, heat transfer mechanisms such as convection and radiation. In the end, a simple model is achieved, but where the physics are still described accurately.

First, the study parameters are formulated, whereafter the results of various topology optimized designs are presented. Secondly, their structural and thermal performances are analyzed to allow for good comparison. For this, a post-numerical nonlinear FE study was performed. This section ends with some general thoughts on three-dimensional optimization for fired-clay bricks, and some further improvements that would allow the algorithm to be used for the design of single-leaf masonry walls with sufficient thermal performances.

3.4.1.1 Study parameters

A reference block (Figure 3.1a) is chosen to better allow for the comparison of the resulting geometries. The design of this block is inspired by an eco-friendly ceramic masonry block by Wienerberger (Thermobrick 15N R+) [14]. Its dimensions are 300 mm long, 140 mm wide, and 190 mm high and the ratio between the net and the gross volume is exactly 50%. These dimensions and volume ratio are then adopted to construct the design domain of the optimization problem (Figure 3.1b). The TO problem is studied in two dimensions. This means that only the horizontal cross-sectional area (300 × 140 mm)

is optimized. The mesh has a size of 600×280 elements, but only the gray area of this design is the layout that can be optimized. The elements on the boundary of the block (8 mm wide) have a fixed density of $x_e = 1$. Ergonomics are considered by enforcing the appearance of a central void. This white region (a rectangular zone of approximately 80 by 34 mm) has a fixed density, where x_e is always 0.



Figure 3.1. (a) Geometry of the reference masonry block (cross-sectional view) and (b) the design domain of the optimization problem (the gray area is to be optimized).

As explained in Section 3.2, the primary objective is the minimization of the thermal compliance. Luckily, the thermal compliance is proportional to the thermal transmittance. In this study, the conductive heat transfer is considered in the Y direction (up-down). Finally, also a minimum member size (internal web thickness) is imposed by adjusting the filter value of the density filter (cfr. Section 2.4.4)

When the thermal compliance is the only objective considered, and no structural constraints are active, no useful results can be obtained. The optimization algorithm struggles to converge, and the material is positioned in random strips (Zebra striping) in the design domain (Figure 3.2a). This can be explained by the fact that mathematically, all solutions with horizontal organizations of the two material phases are equal. Hence, there is no singular optimum that can be found. This problem is partially resolved by activating the fixed densities to the sides and center of the block (Figure 3.2b), however, no meaningful optimization is established.

Therefore, in this study, also the in-plane structural compliance of the block (parallel to the mortar bed) is constrained and by doing so, the lateral stiffnesses of the blocks are optimized at the same time. This makes it possible to include European code requirements concerning minimal strength values and make it possible to manufacture them using existing extrusion processes. According to Eurocode 6 the minimum value for the direct mean compressive strength in the X and Y-direction (parallel to the mortar bed) are respectively 2.0 N/mm² and 5.0 N/mm² [15]. These conditions are considered by measuring the structural compliances under two different load cases, each having two counteracting uniform loads (p_x and p_y) on the boundary of the design domain (Figure 3.3).



Figure 3.2. Preliminary results of the first case study without structural constraints. (a) 'Zebra striping' problem, (b) resulting topology when the fixed densities at the boundaries and at the centre are activated.



Figure 3.3. Design domain with the interacting uniform loads p_x and p_y .

Before the first set of optimized blocks is presented the influence of the penalization value, the filter value, the geometric restrictions, and the external loadings are investigated. Figure 3.4 shows some of the obtained results.

First, it is found that the penalization value is best to be taken around p = 3, as this value gives the best convergence of the optimization process. Additionally, this value is also large enough so that the solutions do not have any remaining intermediate densities. Secondly, in order to prevent the appearance of small internal webs (as in Figure 3.4a and 3.4b), the filter value is set to 6 which (more or less) means that six elements are needed to go from an element with a density of 1 to an element with a density of 0. As such, when used alongside the mesh containing 600×280 elements, the minimum member thickness is equal to 3 mm. After a sufficient number of iterations, this value is again reduced to 1 in order to retrieve a clean (sharp) geometrical boundary.

The solutions in Figure 3.4c and 3.4d show the results when the design is loaded by a series of concentrated loads instead of uniform loads (thus, a concentrated load every few nodes). Although the influence seems interesting, it diminishes when the solid boundary is activated.

Finally, in Figure 3.4e and 3.4f, the ergonomic constraints are activated by enforcing the appearance of a central void and includes the minimum length-scale control of the webs. The relative difference between these two figures is caused by different requirements for the lateral stiffnesses. Figure 3.4f shows a design where the horizontal stiffness (in the X-direction) has been given an increased importance.

3.4.1.2 Post-numerical analysis

In order to verify the performances of the different designs, and because the values of the structural and thermal compliances are too vague, a post-numerical analysis is performed that better estimates the structural and thermal performances with respect to the code requirements. For this, some additional numerical analyses are performed. The applied method for each analysis is described below.



Figure 3.4. Examples of topological optimized rectangular masonry blocks to illustrate the influences of the penalization value, the filter value, geometric restrictions, and external loading.

3.4.1.2.1 The structural performances

An identical loading situation (as elaborated above) is used for the post analysis. However, now a more realistic material model is assumed. Typical masonry blocks are built from fired clay. This material can be regarded as heterogeneous and anisotropic. When combining it with the presence of mortar or grouted joints, the structural behavior is not straightforward [16]. Besides direct compressive failure, the failure of masonry blocks is characterized by spalling, buckling, separation of the shells, and vertical splitting and crushing of the webs [12]. An in-depth study regarding the importance of all these distinct phenomena and specific failure modes lies outside the scope of this PhD. On the other

hand, the structural performance must be studied to some degree of detail. To enable the analysis of the structural efficiency, a nonlinear material model is adopted from the work of Bolhassani et al. [17]. In this study, the heterogeneous and anisotropic properties of masonry block units are simulated by using an Abaqus-compatible concrete damage plasticity model. The used model and its essential values are given in Figure 3.5. Here, σ_{tu} [MPa] is the ultimate tensile strength of the material, and σ_{c0} and σ_{cu} are the initial and ultimate compressive strength respectively. Using this model, the nonlinear response of a masonry unit can be studied. The direct compressive strength is thus defined as the maximum reaction force divided by the gross area of the masonry block. This strength is calculated both in the X and Y-direction, respectively σ_x and σ_y (kN).



Figure 3.5. Parameters of the used material model for uniaxial loading in tension and compression. Values adopted from [17].

3.4.1.2.2 The thermal performances

To determine the thermal performances of the different blocks, the equivalent thermal conductivity is calculated. The law of heat conduction, also known as Fourier's law, states that "the time rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at right angles to that gradient, through which the heat flows". In differential form this is written as:

$$\vec{q} = -\lambda \nabla T \tag{3.14}$$

where, q is the heat flux expressed in W/m², λ is the material conductivity in W/mK, and ∇T is the temperature gradient in K/m.

In this study, we focus on measuring the equivalent thermal conductivity of the structure. For this, we integrate the above differential equation over the total surface (A). In steady-state heat conduction, and assuming isotropic thermal conductivity, the heat flow (Q) between the two boundaries at constant temperature is then given by the following formula:

$$Q = -\tilde{\lambda}A\frac{\Delta T}{\Delta x}$$
(3.14)

Using this equation, we can easily determine the equivalent thermal conductivity $\tilde{\lambda}$. This lambda can be interpreted as the material conductivity of the block as if the complex internal structure was replaced by a homogenous material.

For the calculations in this study, the λ -value of the solid material is set to 0.28 W/mK (fired-clay) and the λ -value of the voids is set to 0.040 W/mK (mineral wool).

Finally, an example of the FE-mesh is presented in Figure 3.6. The mesh seed is 2 mm, and a quad-dominated advancing-front meshing algorithm was used.



Figure 3.6. Example of the FE mesh as used in the post-numerical analyses.

3.4.1.3 Results

The first set of results shows the influence of the load distribution between the load in the X and Y-direction. The optimized geometries are presented in Figure 3.7. The masonry block positioned in the middle (Figure 3.7c) is optimized with an X-Y load ratio equal to one, whereas Figure 3.7a and 3.7b shows the resulting geometries for a block with a higher X-Y load ratio. In contrast, Figure 3.7d and 3.7e have a lower X-Y load ratio. The values of thermal and structural performances based on the numerical study are given in Table. 3.1 and are compared to the values of the reference block. For all load ratio's, the equivalent thermal conductivity and direct compressive strength is given. Numbers typed in bold have a relatively better value in comparison to the reference block. The values perpendicular to the cross-sectional plane (Z-axis) are not shown in the table as they are the same for all solutions because the total net area is identical (buckling of the internal webs is not considered). The equivalent thermal conductivity in the Z-direction is 0.140 W/mK, and the direct compressive strength in the Z-direction is 10.5 MPa.

Figure	p_x / p_y	λ_x (W/mK)	λ _y (W/mK)	σ_x (MPa)	σ _y (MPa)
3.1a	Ref. block	0.121	0.127	4.4	2.1
3.7a	4	0.144	0.102	5.0	1.7
3.7b	2	0.137	0.113	3.9	1.8
3.7c	1	0.127	0.125	2.9	2.6
3.7d	1/2	0.120	0.131	2.9	2.9
3.7e	1/4	0.112	0.137	1.9	3.1

Table 3.1. Comparison of the structural and thermal performances of the optimized blocks to the reference block. Results from the first set of results (as presented in Figure 3.7).



Figure	Reduction of initial stiffness	λ_x (W/mK)	λ _y (W/mK)	σ_x (MPa)	σ _y (MPa)
3.1a	Ref. block	0.121	0.127	4.4	2.1
3.8a	1	0.136	0.116	4.5	2.5
3.8b	2	0.139	0.099	3.7	1.7
3.8c	4	0.129	0.099	2.1	1.7
3.8d	10000	0.130	0.086	1.8	1.1

Table 3.2. Comparison of the structural and thermal performances of the optimized blocks and the reference block. Results from the second set of results (presented in Figure 3.8).

The first aspect that can be observed is that when the load ratio is increased, the thermal performances in the Y-direction improve, as well as having better structural performances in the X-direction. Nevertheless, improving the performances in one direction, negatively influences the performances in the other direction. This means that the requirements for the lateral compressive strengths along with the thermal conductivity in the Y direction are the main important parameters in this study and should be closely checked with code regulations.

The second set of results (Figure 3.8) shows the optimized topologies of masonry blocks for which the structural stiffness in the Y-direction is gradually reduced while minimizing the thermal transmittance across the Y-direction. The idea is that a pareto front can be obtained, and that based on the exact requirements of the lateral compressive strength, the most optimal solution can then be selected. For this parametric study, the resulting topology of Figure 3.8a (which is a slight variation of (Figure 3.7b) forms the starting point of the subsequent parametric study. Figure 3.8b and 3.8c show the resulting geometries for which the values of the lateral stiffness in the Y-direction were decreased by respectively a factor 2 and 4, and Figure 3.8d shows the optimized topology when (almost) no lateral stiffness is required at all. Table 3.2 gives the values of thermal and structural

performances of these geometries in comparison to the values of the reference block. Again, numbers that are presented in bold have a relative better value in comparison to the reference block.

As can be observed, when considering the minimum values presented by Eurocode 8 (see Section 3.4.1.1) the lateral stiffness in the X-direction of the block in Figure 3.8c is still sufficiently strong. Hence, this layout could prove to be a good design for novel thermally efficient masonry blocks.

3.4.2 Case 2: Thermally efficient brickwork support bracket

In this second case study, a brickwork support bracket is studied (Figure 3.9). The design of such bracket is a relevant multi-physics design problem as these are typically made from stainless steel and when poorly designed can reduce the thermal performance of the whole building envelope. The goal of the topology optimization approach is to minimize the heat loss through cold bridging and thus improve the energy efficiency of the bracket. For this, the short cantilever beam problem is used as the design domain of choice. This is another classical benchmark 2D-problem used in many structural topology optimization studies, where one side of a square design domain is fixed and a vertical load acts on the opposite side. Like in the previous study, not only the structural performances are optimized, but also the thermal performances. However, now a different theoretical framework is used, as presented in Section 3.2 and 3.3. A one-material TO study is presented at first; this study analyzes the influence of certain parameters of the structural and thermal interpolation functions used in the TO implementation and discusses the importance of using realistic values for intermediate densities, which can constitute part of the optimized solution. The second study then presents the two-material TO approach for multi-physics optimization and demonstrates its advantages in comparison to the previous approach. Finally, at the end, a real-world brickwork support bracket is optimized using the two-material approach and an optimized design is found.



Figure 3.9. A brickwork support bracket with improved heat transfer characteristics. Source: Adapted from [14].

In this case study, the heat flow through the design domain from left to right (Figure 3.10) is minimized by measuring the thermal compliance of the domain. Additionally, a constraint is put on the design's minimum stiffness, and a limitation is set on the maximum material fraction to be used. An essential aspect of this design problem is that the stainless-steel bracket is fully surrounded by insulation material. Therefore, thermal conduction is the dominant form of heat transfer and allows for the mathematical problem to disregard other, more complex, heat transfer mechanisms such as convection and radiation. In the end, a simple model is attained, where all the physics are still accurately described. The design domain, the boundary conditions, and the external loadings for the problem discussed in this study are shown in Figure 3.10 and the mathematical formulation of the design problem was described in Eq. (3.4). As can be seen on this figure, the mechanical and thermal loads are distributed over 3 nodes at the right-hand side boundary, and a block of 4×4 elements is kept frozen: their density is always 1 and their material properties thus match those of the most rigid material (e.g. stainless steel).



Figure 3.10. The design domain, boundary conditions, and location of external loads for the optimization problem that is presented in case study 2: thermally efficient brickwork support bracket.

3.4.2.1 Study parameters

The units in this study are set with respect to the standard International System; lengths are in millimeter (mm), force units are newton (N), and the Young's modulus is given in mega-Pascal (MPa), whereas the thermal load units are in Watt (W), the thermodynamic temperature is given in Kelvin (K), and the thermal conductivity of a material is programmed in Watts per millimeter-Kelvin (W/mmK). The in-plane dimensions of the design domain are 100 × 100 mm, and its thickness is 1 mm. E_{solid} and λ_{solid} are normalized to unit values, meaning that the maximum material stiffness is 1 N/mm² and the lambda value of a solid element has a value of 1 W/mmK. E_{void} and λ_{void} are varied but their initial value is taken as 1e-3. The total mechanical load (f_{v}) is set to -1 N, and the total thermal load (q) is -1 W (meaning that heat is extracted from the domain). Normalizing these values simplifies the numerical calculation and its mathematical implementation without restraining functionality. The effective compliances can be calculated at any time using the analytical formulas in Eq. (3.14).

$$\tilde{C}_{\rm s} = C_{\rm s} \frac{f_{k,v}^2}{E^0 t}$$
 and $\tilde{C}_{\rm t} = C_{\rm t} \frac{q^2}{\lambda_{\rm void} t}$ (3.14)

The effective compliance is thus proportional to the normalized values of $C_{\rm s}$ and $C_{\rm t}$. Derived from this, a doubling of the load will increase the compliance with a factor of 4. On the other hand, a doubling of the material's maximum E-modulus, the minimum thermal conductivity or thickness will cause a halving of the compliance. In theory, the compliance is also independent of the dimensions of the design domain, as long as the aspect ratio (proportional relationship between the domain's width and height) does not change. Likewise, the number of elements that are used (mesh quality) does not influence this value when the filter radius ($r_{\rm min}$) is mesh-independently defined. Here $r_{\rm min}$ is equal to 2; for a mesh containing 100 × 100 elements. Small numerical errors still remain but these are inherent to the finite element method. However, this error can be disregarded as long as the mesh of the design domain is not taken too coarse.

Finally, the stopping criterion of the optimization process is defined by measuring the maximum change in design variables from one iteration to the next. The main loop is terminated if the change in design variables is less than 2%.

3.4.2.2 Results

3.4.2.2.1 Minimum compliance design

To determine the constraint value of the structural compliance, a traditional minimum compliance design is generated first. The maximum volume of the design (V_{max}) is set to 20% and p_s (structural penalization parameter) and p_t (thermal penalization parameter) in Eq. (3.5) and Eq. (3.6) are set to 3 and 2 respectively. By doing so, an element with an intermediate density will have a relatively lower stiffness while retaining a relatively higher thermal conductivity and is therefore not desirable. This should stimulate black-and-white solutions. The used material properties are presented in Table 3.3.

The resulting optimized design is shown in Figure 3.11. After 185 iterations, the value of the structural compliance (deformability) converges to 38.5 J (Joules) and the value for the thermal compliance (transmittance) is 8.1 J.

Table 3.3. Thermo-mechanical properties of the materials as used in the first numerical study of case study 2.

$\lambda_{ m solid}$	$\lambda_{ m void}$	$E_{ m solid}$	$E_{ m void}$	V
1	1e-3	1	1e-3	0.3



Figure 3.11. Minimum structural compliance design for the short cantilever beam problem with a material reduction of 80%.

3.4.2.2.2 Study 1 – Multi-Physics optimization

Now that we have determined the constraint value of the structural compliance (38.5 J), the multi-physics algorithm can be applied. Of course, it can be expected that when the exact same value for C_s is used, no significant changes should occur. Indeed, almost the exact same optimized structure is found (Figure 3.12a). Running the multi-physics optimization algorithm renders a negligible 1%-increase in thermal performances, caused by the algorithm finding a slightly better local optimum.

In order to improve the design's thermal performances, the structural constraint value of 38.5 J can be modified, or an increase of the maximum allowable volume fraction is required. Because the idea is to keep the same stiffness for the bracket, the latter option is chosen. The results are presented in Figure 3.12. The corresponding values of the optimized thermal compliances and their relative gains are shown in Table 3.4.



Figure 3.12. Maximum thermal compliance designs with $p_s / p_t = 3 / 2$, and for which the maximum allowable volume fraction is varied: (a) 20%, (b) 25%, (c) 30%, (d) 35%, (e) 40%, and (f) 45%.

Table 3.4	Values of	C_{s} ,	C_{t} ,	and	V_f attaii	ned fro	om the	e first	set	of	optimized	solutions	from
Figures 3.	11 and 3.	12.											

Figure	$C_{ m s}$	$C_{ m t}$	Gains	V_{f}	# iterations
3.11	38.5	8.1	-	20%	185
3.12a	38.5	8.1	+0%	20%	200
3.12b	38.5	11.3	+40%	25%	192
3.12c	38.5	13.7	+69%	30%	700
3.12d	38.5	14.9	+84%	35%	807
3.12e	38.5	16.1	+99%	40%	407
3.12f	38.5	16.9	+109%	45%	336

A material addition of only 5% results in a remarkable 40%-increase of the structure's thermal performance. It can also be observed that the relative gains become smaller when

subsequent material is added (Figure 3.13). The maximum volume fraction in the studies that follow are limited to 30% and 40%.



Figure 3.13. Relative gains for the structure's thermal performance for increasing allowable volume fractions.

As mentioned by Bendsøe and Sigmund in [18]: "It is crucial to recognize if a topology design study is supposed to lead to black-and-white designs or if composites can constitute part of the optimal solution." Before, it was mentioned that using $p_s = 3$ and $p_t = 2$ for the thermal interpolation function would stimulate black-and-white solutions because the use of intermediate densities is non-optimal. In agreement, the preliminary study showed good convergence to such a binary structural layout. However, the physical relevance of $p_s = 3$ and $p_t = 2$ is not proven experimentally and is perhaps too disadvantageous in this multi-physics study. Maybe a composite material or microstructure could be created that has better intermediate-density properties. As of today, the correlation between the interpolation curves in multi-physics topology optimization has not been studied in detail. Subsequently, in the following parametric studies, the influence of the penalization parameter p and the bounds on the material

properties are analyzed. It is demonstrated that some values of p_t do not always guarantee black-and-white solutions and that different values of E_{void} and λ_{void} are influential.

In the first parametric study, the parameters p_s and p_t are varied. These *p*-values influence the corresponding interpolation curve and they, at their turn, influence the optimal design solution. Six different variations are presented in Figure 3.14, and their values and performances are given in Table 3.5.



Figure 3.14. Optimal topologies for parametric set 1: (a-c) $V_{\rm f}$ = 30% and (d-f) $V_{\rm f}$ = 40%. Also, $p_{\rm s}$ / $p_{\rm t}$ is indicated below the figure, and $E_{\rm void}$ / $\lambda_{\rm void}$ is set to 1e-3 / 1e-3 for all cases.

It can be noticed that when p_s is larger than p_t (Figures 3.14a and 3.14d), no gray elements arise in the optimized solution. This is because elements with intermediate densities are not beneficial. Their structural performances are weak and their thermal performance worse. On the other hand, the solutions in Figures 3.14c and 3.14f show that gray elements can constitute part of the optimal solution. This can be explained by the fact that these intermediate densities are beneficial for the design's performances. The gray elements have a low stiffness but offer good thermal insulating qualities, and hence

largely reduce the heat flow through the design at a low material cost. They create a thermal break in the design and reduce the heat extraction from the thermal load, while maintaining sufficient structural stiffness. While p = 4 may not be the most realistic interpolation parameter, they do provide intelligent and convincing design solutions.

Figure	$p_{\rm s}/p_{\rm t}$	$E_{ m void}$ / $\lambda_{ m void}$	C_{t}	Gains	$V_{ m f}$
3.14a	3 / 2	1e-3 / 1e-3	13.7	-	30%
3.14b	3/3	1e-3 / 1e-3	17.7	+29%	30%
3.14c	3 / 4	1e-3 / 1e-3	40.6	+196%	30%
3.14d	3 / 2	1e-3 / 1e-3	16.1	-	40%
3.14e	3/3	1e-3 / 1e-3	23.1	+43%	40%
3.14f	3 / 4	1e-3 / 1e-3	57.0	+254%	40%

Table 3.5. Values of p_s and p_t as employed in the first parametric set of solutions and the function values.

In the second and third parametric sets, it is demonstrated that not only the penalization parameter p but also the maximum and minimum values of E and λ impact the design solutions. In other words, the relative difference between the minimum and maximum values of the material properties is studied. In order to do this, only the minimum values are changed. In Figures 3.15 and 3.16, 12 different variations are investigated. Parametric set 2 shows the optimized solutions in which the values of E_{void} and λ_{void} are changed from 1e-3 / 1e-3 to 1e-6 / 1e-2 respectively. In parametric set 3, the values are changed to 1e-2 and 1e-6.

Increasing the minimum value of the thermal conductivity (λ_{void}) while decreasing the minimum value of the Young's modulus (E_{void}) makes the intermediate densities less effective. The gray elements are less stiff in relation to their thermal properties. Nevertheless, these intermediate densities are still included in the optimized solutions and

again form a thermal break (Figures 3.15c and 3.15f). Comparing the results of the optimized solutions with that of the first studied set, the thermal compliances are lower and the gains less prominent (Table 3.6).



Figure 3.15. Optimized topologies for parametric set 2: (a-c) $V_{\rm f}$ = 30% and (d-f) $V_{\rm f}$ = 40%. Also $p_{\rm s} / p_{\rm t}$ is indicated below the figure, and $E_{\rm void} / \lambda_{\rm void}$ is set to 1e-6 / 1e-2 for all cases.

Figure	$p_{\rm s}/p_{\rm t}$	$E_{ m void}$ / $\lambda_{ m void}$	$C_{ m t}$	Gains	$V_{ m f}$
3.15a	3 / 2	1e-6 / 1e-2	12.2	-	30%
3.15b	3/3	1e-6 / 1e-2	15.0	+23%	30%
3.15c	3 / 4	1e-6 / 1e-2	21.1	+73%	30%
3.15d	3 / 2	1e-6 / 1e-2	13.6	-	40%
3.15e	3/3	1e-6 / 1e-2	17.8	+31%	40%
3.15f	3 / 4	1e-6 / 1e-2	27.7	+104%	40%

Table 3.6. Parameters and results from the second studied set of solutions.

In contrast, the results in parametric set 3 (Figure 3.16) apparently have no intermediate densities. The minimum value of E provides enough stiffness for the algorithm to make use of the 'void' material and create small scissions and interlocking patterns in the design. Obviously, these disconnecting white regions create a superior thermal break and subsequently very high values of C_t are achieved (Table 3.7). The question remains how to interpret the results from such optimization studies. Gray elements create efficient thermal breaks, but can a material with these mixed properties be created? Interlocking black-and-white patterns also provide interesting structures to improve the design's thermal characteristics but do not give much certitude that they can perform what was theoretically promised. As mentioned before, the true power of topology optimization is that it is able to produce optimized designs that do not depend on the designer's a priori knowledge. However, in this case, a large number of questions still remain. Different parameters influence the design, and interpretation of intermediate elements is difficult. A multi-material approach is presented in the next paragraph to eliminate these concerns and ambiguities



Figure 3.16. Optimized topologies for parametric set 3: (a-c) $V_{\rm f}$ = 30% and (d-f) $V_{\rm f}$ = 40%. Also, $p_{\rm s} / p_{\rm t}$ is indicated below the figure, and $E_{\rm void} / \lambda_{\rm void}$ is set to 1e-2 / 1e-6 for all cases.

Figure	$p_{\rm s}/p_{\rm t}$	$E_{ m void}$ / $\lambda_{ m void}$	$C_{ m t}$	Gains	$V_{ m f}$
3.16a	3/2	1e-2 / 1e-6	7683.6	-	30%
3.16b	3/3	1e-2 / 1e-6	23944.6	+212%	30%
3.16c	3 / 4	1e-2 / 1e-6	35889.7	+367%	30%
3.16d	3 / 2	1e-2 / 1e-6	37119.7	-	40%
3.16e	3/3	1e-2 / 1e-6	48000.5	+29%	40%
3.16f	3 / 4	1e-2 / 1e-6	60279.2	+62%	40%

Table 3.7. Parameters and results from the third set of solutions.

3.4.2.2.3 Study 2 – Two-material optimization

As demonstrated in study 1, a multi-physics topology optimization approach is easily influenced by even slight changes in the material parameters. Therefore, in this subsection another approach is proposed: once it is clear that intermediate densities could arise, a multi-material approach can be implemented, providing a more robust method to solve the problem. In this study, the two-material approach from Section 3.3 is implemented, thus a second vector of design variables is added to the optimization process. The goal is identical to the previous study. However, this approach should eliminate the appearance of gray elements and provide more practical design solutions. In the following study, the material properties of the 'void' material and the stiffest material (solid – material 2) are kept constant while the thermomechanical properties of the extra material (solid – material 1) are varied (Table 3.8).

The results of the study are shown in Figure 3.17. The black pixels or elements represent the presence of the stiffest material, with properties E_2 and λ_2 (steel or aluminum). While the white pixels represent the soft material with E_{void} and λ_{void} (insulating material), the presence of the extra solid material 1 (E_1 and λ_1) is illustrated by use of a color. The different colors represent different materials such as rubber, polyester, or polyamide. In all cases, material 1 has a lower stiffness than material 2, but has a lower thermal

conductivity. As before, the constraint on the structural compliance is limited to 38.5 J and the maximum volume fraction is 30% or 40%. This volume constraint only affects the first vector of design variables, the variables that control the relationship between the soft and the solid material (1 or 2). The ratio between material 1 and 2 is not controlled in any way by the algorithm. It can thus choose freely to use the material it deems most optimal. The difference in material properties between material 1 and 2 is based on real materials. As can be seen on Figure 3.17, three colors are used: red, green, and blue. The 'red' material is based on the material properties of linearized rubber versus steel; the 'green' material is based on the difference in properties between polyamide and aluminum. E_{void} and λ_{void} are set to 1e-6, and the SIMP penalization parameters that are used are $p_s = 3$ and $p_t = 2$.

	<i>E</i> ₁ / <i>E</i> ₂ (N/mm²)		<i>E</i> ₁ / <i>E</i> ₂ (N/mm ²)
Red	100 / 210000	>>>	0.0005 / 1
Green	2500 / 210000	>>>	0.0119 / 1
Blue	1300 / 69000	>>>	0.0188 / 1
	λ_1 / λ_2 (W/mK)		λ_1 / λ_2 (W/mmK)
Red	0.15 / 50	>>>	0.003 / 1
Green	0.20 / 50	>>>	0.004 / 1
Blue	0.28 / 205	>>>	0.001 / 1

Table 3.8. Thermo-mechanical properties of the multi-material optimization study

The results of the multi-material study (Table 3.9) clearly show that for a volume fraction of 30% only the green and blue material is positioned to act as a thermal break. The other material (red) is too weak to be used as a thermal break and still satisfy the structural stiffness constraint. The optimized solutions with a volume fraction of 40% show similar results. Again, the green and blue materials are creating a thermal break. However, it must

be mentioned that since aluminum has a much higher thermal conductivity than steel, it is more beneficial to use the extra material (in this case polyamide) and the gains are easier to achieve in comparison to the other examples. Therefore, the solutions of the blue designs cannot be compared 1 to 1 with those of the red and green designs.



Figure 3.17. Optimized designs for the multi-material topology optimization study. The 'red' material is based on the material properties of linearized rubber versus steel; the 'green' material is based on the difference in properties of polyester versus steel; and the 'blue' color is created based on the difference in properties between polyamide and aluminum.

3.4.2.2.4 Study 3 – Optimized support bracket

Following the numerical studies performed in previous subsections, this part discusses a more realistic case study example, namely, the optimization of the original brickwork support bracket as presented in Figure 3.10. The problem formulation is similar to that of the previous studies. However, now the boundary conditions and the loads are adopted more closely to the real product (Figure 3.19) and the material properties are also not normalized.

Figure	E_1	λ_1	C_{t}	Gains	$V_{ m f}$
3.12c	-	-	13.7	-	30%
3.17a	0.0005	0.003	26.9	+96%	30%
3.17b	0.0119	0.004	36.2	+164%	30%
3.17c	0.0188	0.001	94.9	+593%	30%
3.12e	_	-	16.1	_	40%
3.17d	0.0005	0.003	29.0	+80%	40%
3.17e	0.0119	0.004	43.2	+168%	40%
3.17f	0.0188	0.001	188.3	+1070%	40%

Table 3.9. Results of the optimization study 2 compared with the results from Table 3.4.

According to Halfen, the design illustrated in Figure 3.9 is a relatively newly developed brickwork support bracket with improved heat transfer characteristics [19]. Also, the Halfen HK5 Anchor has a higher load capacity and requires less steel than its predecessor, while reducing the effect of thermal bridging. The maximum vertical mechanical load is 4 kN and is located 60 mm from the edge, introducing a bending moment. The thermal loads are also attached on the left edge of the design domain. The material properties for *E* and λ are presented in Table 3.10. In this study, the properties of the extra material are based on ABS plastic and the insulating material is mineral wool. The material thermal conductivities are presented in W/mK instead of W/mmK.

Color	Material	E (N/mm ²)	λ (W/mK)
Black	Steel	210,000	50
Green	ABS	3500	0.15
White	Mineral wool	0.01	0.044

Table 3.10. Thermomechanical properties of the materials as employed in the case study.



Figure 3.18. The brickwork support bracket design from Halfen [19], and the interpreted design domain, loads, and boundary conditions.

First, a minimum structural compliance design is generated with a maximum volume fraction constraint of 20% (Figure 3.19a) Secondly, a multi-physics optimization is performed that maximizes the thermal compliance (Figures 3.19b and 3.19c). And finally, a multi-material optimization is also performed (Figures 3.19d, 3.19e and 3.19f). The structural and thermal performances of all the different solutions are presented in Table 3.11.

As expected, the thermal performance improves when more material is available. The last design has the highest and overall best score and is characterized by two ABS thermal breaks. In this design, the green zones fully disconnect the supports from the thermal loads which aids in the reduction of the heat flow. However, the cost of ABS relative to stainless steel is not incorporated in the optimization. Now, each material has the same cost function (volume). Nevertheless, the gains for all solutions are quite good in comparison to the initial design. In particular, the first optimized design already provides satisfying gains.


Figure 3.19. Optimized designs for the brickwork support bracket using the different optimization strategies (white = mineral wool, black = steel, and green = ABS plastic).

Table 3.11. Structural and thermal performances of the optimized designs for the brickwork support brackets.

Figure	$p_{\rm s}/p_{\rm t}$	$C_{ m s}$	C_{t}	gains	$V_{ m f}$
3.19a	3 / 2	3.06	3.90	-	20%
3.19b	3 / 2	3.06	4.22	+8%	30%
3.19c	3 / 2	3.06	4.29	+10%	40%
3.19d	3 / 2	3.06	4.33	+11%	30%
3.19e	3 / 2	3.06	4.38	+12%	40%
3.19f	3/2	3.06	4.40	+13%	50%

3.4.2.2.5 Convergence plots

In this subsection, the convergence of the objective and constraint functions are discussed (Figures 3.20 and 3.21). For this, the convergence plot (from the result in Figures 3.18e) was chosen as representative for most design solutions as they converged very similar for all results (often within 100 iterations). It is observed that the convergence of the structural compliance occurs quickly and is stable during subsequent iterations. Similarly, the convergence of the volume constraint is extremely swift. Concerning the thermal compliances, the convergence throughout all optimization studies is a bit more unstable with a sudden ramp arising when thermal breaks are created. The appearances of these thermal breaks in the solution are indicated a with vertical (red) dash line. In one exceptional case (Figures 3.21), it takes the optimization algorithm almost 300 iterations to converge.



Figure 3.20. Convergence plot of the volume fraction, the structural, and the thermal compliance of the solution presented in Figure 3.17e.



Figure 3.21. Convergence plot of the volume fraction, the structural and the thermal compliance of the solution presented in Figure 3.17c.

3.5 Discussion

In this chapter, topology optimization was performed taking into account simultaneous structural and thermal analyses. The design of a thermally efficient masonry block and a brickwork support bracket were addressed. In both studies, the goal was to reduce localized thermal heat flow (cold bridging) while retaining sufficient stiffness. A number of problems related to multi-physics topology optimization were examined and the benefits of a two-material implementation were presented. The aim was to improve the robustness of such multi-physics optimization processes and provide a method to accurately determine beneficial design solutions.

3.5.1 Case 1: Thermally efficient masonry block

In the first case study, it was shown that topology optimization can generate a large diversity of different material distributions, where the results are heavily influenced by the initial boundary conditions and study parameters. Attempts were made to explore new design ideas and open the existing mindset of thinking outside the box. Additionally, the subsequent analyses have shown that some distributions are more favorable than others. Therefore, studying the topology-optimized results could help to give a clue to why certain material distributions are preferred and others not so much.

An advantage of the approach is that TO could potentially aid engineers and designers to understand better the underlaying mathematical physical problem and to see possible ways to optimize their design. A drawback of the approach was the need for postnumerical validation. Unfortunately, this strengthens the current practice where TO is mainly used for the conceptual design phase.

Additionally, this first case study has mainly focused on the two-dimensional TO of masonry blocks, where only the 2D cross-sectional area was considered. The idea behind this reasoning came from the fact that most existing masonry blocks are characterized as being two-dimensionally extruded perforated blocks. As such, it could be argued that a three-dimensional topology optimization approach would not provide the designer with any additional information. But this is too short sighted and forgets to take into account the fact that bricks are not made of linear elastic material. As such, buckling and spalling of the vertical webs is of major concern. Also, with regards to the rise of new production methods as discussed in the introduction section of this doctoral thesis, new manufacturing methods could transform the production process of these blocks, where the shape of a masonry block is no longer forced to be 2D-extruded.

An exploration of an approach where a brick is optimized in three dimensions, is shown in Figure 3.22. For this example, the Abaqus FE solver and optimizer was used. In this preliminary study, a force-controlled loading was used, and the brick was optimized to structural loads only. Unfortunately, in this analysis no thermal objective was yet defined.

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Also, nonlinear failure modes such as buckling and spalling of the vertical webs were yet not included., and no manufacturing constraints were considered. The result shows a shape-complex geometry that leaves room for further investigation.



Figure 3.22. A three-dimensionally optimized masonry block cut in half.

Comparative discussion

To conclude this subsection, a comparative discussion is given to compare the results from this work with the findings found in the work of Bruggi & Taliercio. [13]. Both frameworks are very similar ([13] was also referred to at the start of the methodology section of this case study). However, the initial problem formulation is not completely identical (e.g., the use of different design domains, application of a distinct interpolation model...). In this final discussion, the similarities and differences are briefly presented.

As also found in the study by Bruggi & Taliercio, maximizing only the thermal properties of the masonry blocks leads to unfeasible designs: "a bulky inner core, with negligible out-of-plane stiffness". Similar findings could be observed in this study (see Figure 3.23). Furthermore, Bruggi & Taliercio also studied a conventional staggered design, and for the same value of the lateral compliances, an optimized layout was generated. They registered, a decrease in transmittance of about 5% for the optimized block. Comparing this value, to the optimization of performances in this case study, as also shown in Figure 3.7c and the reference block of Figure 3.1a, a slightly lower improvement was achieved here. A 2% increase of performances was obtained. Finally, both studies do agree on the fact that the,

(quoting Bruggi & Taliercio): "optimal material distributions obtained can be exploited to achieve the pre-design of new and non-standard types of blocks".



Figure 3.23. The design of masonry blocks with enhanced thermomechanical performances by Bruggi & Taliercio [13]: (a) a conventional staggered design, and (b) the optimized layout achieved for the same values of non-dimensional compliances.

Another comparable follow-up study was performed by Ganobjak & Carstensen [20], made public after the publication of the study discussed in this work. This paper also presents a similar approach towards using topology optimization to improve the properties of bricks and blocks. However, here also the use of silica aerogel filling is discussed. Again, very similar results were obtained. Ganobjak & Carstensen additionally demonstrated that the optimized bricks with aerogel filling have a much smaller U-value regardless of the bricklaying direction. Thus, it was found that much thinner walls could be constructed with the same thermal behavior.

3.5.2 Case 2: Thermally efficient brickwork support bracket

In the second case study, the preliminary results showed that a small increase of the maximum allowable volume fraction could largely improve the thermal performance while retaining equal stiffness. A small additional cost of raw materials thus proved to be very

beneficial in these kind of design problems. Further on, the first parametric study revealed that a combined structural and thermal topology optimization approach can cause serious confusion regarding material characterization. The penalization parameters that are used to construct the interpolation curves for the thermal density-conductivity relation are hard to determine and when choosing improbable parameters, interesting design solutions could be overlooked. To solve this problem, a multi-material topology optimization approach was included which added an extra set of design variables. In this way, a new type of material could be added with predefined properties. Using this multi-material, multi-physics approach, innovative and new topological solutions could be found. Choosing the right materials remains important, but unlike the traditional approach, the design solutions are unambiguous, and a realistic optimum could be obtained. The final study then optimized a brickwork support bracket inspired by a design of Halfen and an improved design proposal was found that included two ABS thermal breaks.

Future research could focus on experimenting with more than one additional material and try to include a cost function to incorporate the price variance of different materials. Also, the incorporation of nonlinear material models into the algorithm could become relevant and the fact that the design of construction-related components is often affected by, not one, but many different disciplines should be taken into consideration. For example, in addition to structural and thermal performances, other aspects such as acoustic, fire safety or durability and long-term serviceability could be added.

Another important question that remains is to which extent multi-material optimized solutions can already be realized in practice. Certain issues could be the material strength, the strength of the bonding between different materials, and manufacturing constraints that limit the production of the highly complex shapes. Also, as discussed in [21], joining or bonding a material with another has a certain cost. How can this be implemented in the algorithms? A first idea for a joint optimization as part of the topology optimization has been presented in [22], but still, much more research will be required to - not only provide theoretical optimized solutions - but also realisable solutions.

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One of many possible solutions to the bonding problem is elaborated on in the next chapter presenting a single variable-based multi-material structural optimization approach.

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CHAPTER IV

TOPOLOGY OPTIMIZATION & ADDITIVE MANUFACTURING

This chapter is partially adopted from the following journal publications. The copyright of the original publications is held by the respective copyright holder, see the following copyright notice:

1. G. Vantyghem, M. Steeman, V. Boel, and W. De Corte, "Multi-physics topology optimization for 3D-printed structures," in Proceedings of the IASS Symposium 2018, Boston, USA, 2018.

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2. G. Vantyghem, W. De Corte, M. Steeman, and V. Boel, "Density-based topology optimization for 3D-printable building structures," Structural and Multidisciplinary Optimization, vol. 60, no. 6, pp. 2391–2403, 2019.

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4.1 Introduction

Some developments on multi-physics topology optimization for building components were already presented in Chapter 3. First, a framework was proposed for the combined optimization of both mechanical and thermal aspects of a design. Secondly, a multi-material TO study was presented. The idea of the multi-material approach was to eliminate uncertainties regarding the interpolation parameters. As such a strict number of materials could be selected beforehand and from which the optimization process could determine the most ideal material distribution. This chapter presents an additional framework and focuses on bringing together two (e)merging technologies that show great potential for realizing highly efficient building structures: (i) topology optimization for simulation-driven design and (ii) additive manufacturing to produce the resulting complex optimized shapes. For this, a dedicated link between the two is made.

Instead of aiming towards black-and-white TO solutions, the idea here is to allow for intermediate densities to remain present in the optimized solution. The reason for this is that intermediate densities (with 'in-between' material properties) could prove to be beneficial when used in multi-physics design problems. The concept of using the full range of material densities is based upon the ability of 3D printing to carefully control two mutually exclusive aspects of a produced part: the exterior walls (or perimeter) and its infill. The infill is the material that occupies the internal part of the final product. Typically, the percentage of the infill can be modified from 0% (a hollow part) to 100% (totally solid part). By linking this infill density to the original vector of design variables of the TO problem, a complete physical representation of the density variables is made. Nevertheless, the exact multi-physics material properties should be determined with care.

In this chapter, the opportunities for such density-based topology optimization with 3Dprintable infill structures are presented. A general framework is constructed, and its challenges with regards to the thermal material characterization are addressed. A preliminary design problem is optimized and demonstrates its potential. Also, some problems regarding the material characterization are addressed. A next section shows how

a new penalty scheme can be constructed by investigating the homogenized properties of the infill patterns, and a complete new set of interpolation functions is created based on the homogenized properties of a triangular pattern. This chapter ends with a topology optimization study, performed using the GCMMA algorithm, and shows the principle of using a weighted-sum dual objective. One part of the equation will aim to maximize stiffness, while the other attempts to minimize the thermal transmittance. A case study is presented to demonstrate the effectiveness of this novel multi-physics optimization strategy. Results show a series of optimized topologies with a trade-off between structural and thermal efficiency.

4.2 General framework

With the rise of 3D printing, a different approach can be taken where the design variables (the density of each element) are being linked to 3D-printed infill densities. 3D-printed infill patterns are often used to further reduce weight and material cost [1]. The strength and stiffness of an object is then linked to the infill pattern and its density. Some parts in a structure may benefit more from a high-density internal structure, while in other parts this would be a waste of resources. Additionally, some studies have shown that 3-dimensional infill patterns affect thermal performances by encapsulating air [2], and in [3] this is being investigated to inspire and create new insulation materials.

In contrast to the framework of the previous chapter, this study will start by analyzing the specific mechanical and thermal material properties of such infill patterns and use this information to produce the new interpolation schemes. In this way, a new kind of multimaterial optimization can be performed without additional cost of adding a new vector of design variables for each additional material. One requirement is of course that the interpolation schemes correspond to the physical properties of the infill pattern. In this work, a numerical homogenization approach therefore is used.

4.2.1 Conceptual exploration

As discussed before, the basic principle behind topology optimization with continuous design variables is that the material densities can attain any value between zero and one: zero meaning void or empty space and one meaning that material is present. In order to achieve a zero-and-one design (also referred to as black-and-white or solid/empty topology), the material properties are being penalized for intermediate values. In this way, it is better for the design algorithm to remove the intermediate 'gray' elements in favor of a design with only 1's and 0's. A benefit of such resulting topology is that it is easy to extract the final design and allows for straightforward manufacturing. This idea of material penalization is not a purely mathematical practice to enforce these black-and-white topologies but originates from the physical relationship between material properties and density in composite materials [4]. For example, within a structural design problem and using the SIMP method, a *p*-value of at least 3 means that the interpolation scheme is in accordance with the Hashin Shtrikman (HS) bounds on material properties (as also discussed in [4]). However, at some points on the interpolation curve, the most optimal (stiffest) material properties are not yet reached. In accordance, other schemes have been suggested [5,6], such as RAMP (Rational Approximation of Material Properties) and SINH (named according to the hyperbolic sin function: 'sinh'), where material properties correspond better with the actual behavior. Nevertheless, these were mainly constructed for mechanical problems only and are not valid for multi-physics problems. Additionally, in this chapter, the influence of heat transfer by convection and radiation in the infill patterns is investigated.

The basic concept for the framework is as follows: the design variables are being linked to the different realizable densities of a 3D-printed infill pattern (see example in Figure 4.1). A high density represents a solid structure with high stiffness, and certain thermal properties (but mostly weak), whereas an intermediate density provides the structure with improved insulation qualities, because air gets trapped in the small internal voids of the infill pattern. On the other hand, when the infill pattern's density approaches 0, the air cavities become much larger as such that the heat flow by convection and

radiation becomes relevant. This decreases the thermal performances of the infill structure. A simplified interpolation scheme that shows this concept is presented is Figure 4.2 and its equations in Eq. (4.1). Here $x_e = 0.5$ has the optimal thermal performances.

It can be observed that the design variables clearly have three extreme (optimal) states. One state symbolizes free flowing air (where, $x_e = 0$), another state represents the solid structure ($x_e = 1$). Finally, a third extreme state is observed at $x_e = 0.5$, which symbolizes a thermally efficient infill structure.



Figure 4.1. Different 3D-printed infill densities (material percentage) for a grid pattern. Source: manufactur3dmag.com

$$E_{e}(x_{e}) = E_{\min} + x_{e}^{3}(E^{0} - E_{\min})$$

$$\lambda_{e}(x_{e}) = \lambda_{\min} + \begin{bmatrix} 1.75 \ (0.5 - x_{e})^{2} \\ + 3 \ (0.5 - x_{e})^{3} \\ + 3 \ (0.5 - x_{e})^{4} \end{bmatrix} (\lambda^{0} - \lambda_{\min}) \qquad (4.1)$$

$$V_{e}(x_{e}) = x_{e}$$

The exact mathematical validity of this interpolation scheme is of course not yet determined. For this, the true material properties of the infill pattern should be analyzed in detail. Section 4.3 will present a method for doing so. Nevertheless, a first preliminary

design study is performed to better grasp the importance and opportunities of this alternative penalization scheme for multi-physics TO problems.



Figure 4.2. Simplified formulation of an interpolation scheme for multi-physics topology optimization with 3D-printable infill structures (according to the proposed equations in Eq. (4.1)).

4.2.2 Preliminary pilot study

4.2.2.1 Study parameters

This pilot study presents a 2D design domain, representing a fictitious roof structure for a - to be 3D-printed – polymer-based pavilion. The roof structure is supported on its edges and is loaded by a distributed vertical load on the top and bottom surface. The boundary conditions and mechanical loadings for this problem are shown in Figure 4.3a. Then, the design domain is optimized for maximum stiffness and minimal thermal transmittance. More correct, two objectives are used, the first objective minimized the structural compliance, while the second maximizes the thermal compliance.

The design domain contains 120×1200 square unit elements (1 \times 1 mm), the equality constraint for the volume fraction of the optimization procedure is set to 50%, and the

value of the density filter is set to 4.8 mm. The loads of the problem are normalized, as well as the material properties. This means that the solutions might not be mathematically exact. However, the primary goal of this first pilot study is to investigate the validity of the constructed interpolation schemes, as presented in Eq. (4.1). A more realistic formulation of the same problem is presented in Section 4.3 and a complete version of the problem formulation is given in Section 4.4.1.

The optimized topology that only considers the first objective is presented in Figure 4.3b. A second optimized result is shown in Figure 4.3c that only considers the second objective. A beam with the smallest thermal transmittance (U_{mean}) is created. The voids are presented in blue, the solid material in red and the intermediate (thermally efficient) material is displayed in green. As can be noted, the optimized solution in Figure 4.3b does not include any of this green, thermally efficient material. A very stiff frame-like structure is created. In contrast, the solution presented in Figure 4.3c only contains the thermally efficient material; a 'green' beam is created. In this example, the maximum material fraction was set to 50%. In other words, a maximum of 50% of the 'red' material can be used. However, because the thermally efficient material has a density of 0.5, for every element of solid material, two elements of 'green' material can be used. Hence a full beam of 'green' material is achievable.

The beam's deflection (δ_{max}) and its U-value (U_{mean}) are presented below the figures. The maximum and minimum values for the Young's modulus were taken as 2400 MPa and 2.4e-6 MPa and for lambda, values of 1 W/mK and 0.02 W/mK were adopted respectively. Because the solution in Figure 4.3b is the stiffest solution, and the beam in Figure 4.3c is the most thermally efficient solution, their values form the outer limits of the optimization problem.

Figures 4.4a and 4.4b now present two solutions from the multi-physics optimization study with a weighted-sum multi-objective. The differences between these originate from a difference in the importance of each weighting factor (w) in the objective where w_1 influences the weight of the structural compliance, and w_2 impacts the importance of the thermal compliance (Eq. 4.2).

$$\min_{x} \qquad w_1 \frac{C_s(\mathbf{x})}{f_1} - w_2 \frac{C_t(\mathbf{x})}{f_2} \tag{4.2}$$

The parameters f_1 and f_2 are additional scaling parameters that are used to normalize the original values (i.e., a scaling operation is performed) to make both compliances dimensionless. f_1 and f_2 are initially set to 30000.



(c) δ_{max} = 10.52 mm / U_{mean} = 0.16 W/m²K

Figure 4.3. (a) The design domain, boundary conditions, and external loadings for the preliminary TO study and its results for (b) maximum stiffness design and (c) most thermally efficient design.



Figure 4.4. Results of the multi-physics topology optimization problem for the preliminary study.

Figure 4.4a clearly presents a topological solution that could not have been designed by any human designer. The resulting topology is a complex mix of the different types of density. A large portion of the beam is made from the 'green' material, strengthened by the 'red' material at the most vital locations. The result nicely demonstrates the benefit of a multi-physics topology optimization; a material distribution can be created that exactly finds the right trade-off in relation to the designer's objective. In comparison with the stiffest design from Figure 4.3b, the beam has lost some of its stiffness (deflection: +26%), but its U-value has improved radically (-59%). The second solution (Figure 4.4b) was generated by applying a larger weight to the second term of the weight-sum objective. The weight factor of the thermal compliance was increased.

An interpretation of how these resulting topologies influence the settings of the 3D-printed component is given in Figure 4.5. In this example, the open-source slicing software 'Cura' was used to transform the solution from Figure 4.4a into a printable structure with its associated infill density for the intermediate value of x_e .



Figure 4.5. Physical representation of the optimized solution presented in Figure 4.4a, sliced by the slicing software Cura.

4.2.2.2 Pareto set of optimized results

The previous section stated: "the result nicely demonstrates the benefit of a multi-physics topology optimization; a material distribution can be created that exactly finds the right trade-off in relation to the designer's objective". However only two pareto-optimal solutions were shown. In this section, a complete set of pareto-optimal results is generated and discussed along with their position within the front. For this study, a set of eleven different weighting factors of the objectives functions were chosen, and the various results recorded. Equally spaced weighting factors were selected (Table 4.1). The first graph (Figure 4.6) shows the pareto front with respect to both objective functions and the second graph (Figure 4.7) presents the pareto solutions with respect to their maximum mid-beam (bottom) deflection and mean thermal transmittance.

Fig. 4.6 & 4.7	w_1	<i>W</i> ₂	# iterations
А	5	30	ND
В	7.5	27.5	ND
С	10	25	ND
D	12.5	22.5	ND
E	15	20	3691
F	17.5	17.5	2485
G	20	15	ND
Н	22.5	12.5	1691
Ι	25	10	1211
J	27.5	7.5	1823
К	30	5	868

Table 4.1. Weighting factors and number of iterations that were needed as used in the multi-objective TO problem.



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front. The vertical axis shows the mean U-value, and on the horizontal axis, the mid-span deflection is shown.

A first aspect that can be observed is that equally spaced weighting factors do not automatically generate equally spaced solutions on the pareto front (this problem was briefly discussed in Section 2.5.5). Nevertheless, a sufficiently dense population of the front is achieved in order to discuss the shape of the frontier. The shape of the Pareto front is found to be neither convex nor concave. An 'S' shaped form of the trade-off frontier is discovered. This means that, at either ends of the front, a large sacrifice is needed in one objective in order to improve the other objective only slightly. 'S' shaped trade-off frontiers usually offer some preferred points. For practical engineering considerations, a second graph (Figure 4.7) was constructed. The pareto front is now convex and some critical knee point solutions can be identified (i.e., a solution on the Pareto front that requires a large sacrifice in one objective to improve the other objective). The core idea is that by studying the pareto front, this might greatly help the user in his/her decisionmaking process for choosing the final solution, as it provides the user with additional domain knowledge. For example, allowing for a small increase of the structure's deflection, might be quite beneficial for improving its thermal transmittance.

4.2.2.3 Extension

The idea to optimize large-scale building components (such as the roof structure) can be further extended. Instead of optimizing just the roof element of a to-be 3D-printed pavilion, also the complete building envelope can be included in the design domain of the optimization procedure. An illustrative example of how such optimized design solution could look like, is presented in Figure 4.8. In this figure, the inner free volume is fixed, where no structural supports can be positioned, and the remainder of the domain can be optimized freely (above and around this fixed area). The potential problem could be formulated as follows: the volume fraction of the structure is to be minimized with respect to certain minimal requirements for the structural and thermal performances (as illustrated on the figure itself).

4.2.2.4 Conclusions

With this general framework and the presented pilot study, the benefit of using 3D-printed infill patterns within TO is demonstrated. However, not much attention was spent on the development and verification of the implemented interpolation scheme. In fact, the equations for the scheme were probably too beneficial, hence their preference for converging towards only three optimal states. The following section will, therefore, show how the true and realistic material properties of such infill patterns can be determined.



Figure 4.8. Multi-physics topology optimization of a dome structure presented as a purely conceptual design problem where the element density variables have three optimal states. $x_e = 0$ (void), $x_e = 0.5$ (mesostructure), and $x_e = 1$ (solid).

4.3 Accurate penalization scheme for 3D-printable structures

4.3.1 Numerical homogenization of infill patterns

This section will now present how the realistic material properties can be determined, and how new interpolation schemes can be constructed using numerical homogenization. For each material characteristic, a unique interpolation function is determined. A first relation is made between the design variables and their stiffness, $E_e(x_e)$. Secondly, a match is found for their thermal insulation quality, $\lambda_e(x_e)$. And finally, a link also exists between the density and its volume, where $V_e(x_e)$ mostly depends linearly on **x**.

For this characterization, numerical homogenization techniques are used. Numerical homogenization is an efficient method to acquire the effective macroscopic properties of periodic composite materials. Only the unit cells (Figure 4.9) must be defined, and these are periodically repeated by the code into the 2D space. By assigning an extremely low Young's modulus for the void regions, a single-phase cellular material can be represented. For this analysis, a MATLAB code from Andreassen and Andreasen [7] is used.

The 3D-printed infill pattern that is chosen for this study is a pseudo-isotropic triangular pattern (Figure 4.10), meaning that their linear elastic properties are very close to uniform in all orientations. Patterns that share this property are honeycomb patterns, tri-hexagonal patterns, or the 3D gyroid pattern. With this model, both the structural as well as the thermal material properties are calculated and compared to the SIMP and RAMP model. Additionally, the Hashin Shtrikman (HS) bounds [8,9] are given as well. These bounds are well-known in the theory of composites, as they represent the extreme and effective properties of isotropic two-phase composites. In the limit when the properties of one of the phases (voids) are equal to zero (single-phase cellular material), the HS upper bounds are defined as

$$\frac{E^*}{E^0} \le \frac{x}{3-2x} \tag{4.3}$$

for Young's moduli, and

$$\frac{\lambda^*}{\lambda^0} \le \frac{x}{2-x} \tag{4.4}$$

for the thermal conductivity. For the structural homogenization, this (single-phase) simplification can be made because the stiffness of the voids (air) is indeed (almost) zero. For the thermal interpolation model, the simplification of Eq. (4.4) is not justified as the material properties of the voids are not zero. Therefore, its extended version Eq. (4.5) should be used, which is mathematically equivalent to the two forms of the well-known Maxwell–Eucken model [8].

$$\lambda^* \le \frac{2\lambda_{\min} + \lambda^0 - 2(\lambda_{\min} - \lambda^0)x}{2\lambda_{\min} + \lambda^0 + (\lambda_{\min} - \lambda^0)x}$$
(4.5)

In these equations, E^* and λ^* are the (effective) Young's modulus and thermal conductivity of the composite, and E^0 , λ^0 , and x are the Young's modulus, thermal conductivity, and volume fraction of the solid phase material respectively. Finally, λ_{\min} is the thermal conductivity of the void regions in the microstructures and is given the value of air at atmospheric pressure at 10°C ($\lambda_{\min} = 0.025$ W/mK).



Figure 4.9. Unit cell topologies with different infill densities (from left to right: 0.14, 0.28, 0.40). The unit cell is finely discretized by a mesh of 420 × 420 elements.



Figure 4.10. PLA bricks printed with different triangular infill densities created on an Ultimaker 3. From left to right: 0.18, 0.22, 0.33.

4.3.2 Structural material properties

Although the homogenized material properties can be deduced for any type of infill pattern or microstructure, the presented interpolation model was here determined for a triangular pattern made of PLA plastic. Some initial assumptions are given. The elasticity, or Young's Modulus, of the PLA (E^0) is set to 2500 MPa, while the voids (air) are set to a very low Young's modulus of 0.001 MPa. The Poisson ratio (v^0) of the material is 1/3, and a plane stress relation is used. After the numerical analysis, the MATLAB code provides the user with a homogenized stiffness matrix from which the following material characteristics can be deduced: E_x , E_y , G_{xy} , v_{xy} and v_{yx} . Due to the pseudo-isotropy of the pattern, the Young's moduli in both normal directions are approximately equal and a link exists with the shear modulus (G). Finally, simplifying v_{xy} and v_{yx} to v^0 , the Young's modulus of the homogenized material is calculated as the average of E_x , E_y , and 2 (1+ v^0) G_{xy} .

Figure 4.11 presents the results of the numerical homogenization study for a range of different element densities. They are presented as circles on the graph and compared to several experimental results (triangles). The size of the experimental samples is $50 \times 50 \times 20$ mm, and the samples were 3D printed in white PLA plastic on an Ultimaker 3. Each density was printed four times. Two samples were printed with a nozzle width of 0.4 mm

and another set was printed with a nozzle width of 0.8 mm. A uni-axial compression test was performed under displacement-controlled loading (speed: 5 mm/min). As can be seen in the graph, a good fit between both results exists. The HS upper bound is displayed by a dashed line, while the SIMP model (power law) function is shown by a dotted line. For use in the topology optimization study, the best fitting curve is obtained using a RAMP (Rational Approximation of Material Properties) model with a *q*-value of 2.6. The mathematical function used in the topology optimization study is therefore given by the following equation:

$$E_{e}(x_{e}) = E_{\text{void}} + \frac{x_{e}}{1 + q(1 - x_{e})} (E^{0} - E_{\text{void}})$$
(4.5)

w:
$$E_{\text{void}} = 0.001, E^0 = 2500, \text{ and } q = 2.6$$



Figure 4.11. Results of the experimental and numerical study compared to the RAMP and SIMP model, and the Hashin-Shtrikman upper bound for an isotropic material with Poisson ratio 1/3 mixed with void.

4.3.3 Thermal material properties

The numerical homogenization for the calculation of the thermal conductivities is very analogous to that of the elastic problem, though it is enough to solve for a scalar field – the temperature. Since air becomes trapped in the internal voids, the infill pattern largely increases the material's thermal insulation quality. Logically, the lower the element's density, the lower its thermal conductivity will be. The thermal conductivity of the solid PLA (λ^0) used in this study is 0.275 W/mK, while the thermal conductivity of the cavity air (λ_{min}) is set to 0.025 W/mK. Figure 4.12 presents the results of the numerical homogenization study. As can be seen, the numerical data points are close to the HS upper bound, and the best fitting RAMP curve is found for a *q*-value of 0.9.



Figure 4.12. Results of the numerical homogenization study compared to a RAMP model, a SIMP model, and the Hashin-Shtrikman upper bound for an isotropic material mixed with air cavities.

Because the MATLAB code by Andreassen and Andreasen only takes into account heat flow by conduction, the air inside the cavities is regarded as being completely stationary. This simplification is an inaccurate assumption. The thermal performances will be influenced by convection and radiation inside the air cavities. To cope with this problem, an additional analytical study is carried out conform EN ISO 10077-2 [10]. The cavities of the infill pattern are considered unventilated, and the emissivity of the PLA is set to 0.90. Because the simplified analytical equations for convection and radiation in EN ISO 10077-2 are only valid for rectangular cavities, the triangular voids are considered as if they were rectangular (with identical area). This assumption is made to avoid additional and more complex fluid dynamics calculations. Also, a link is made between the infill pattern density and the nozzle width. Because the focus of this work is on large-scale 3D printing applications, the nozzle sizes that are included in this study are: 4 mm, 2 mm, and 0.8 mm. For each of the nozzle sizes, the corresponding line distance is extracted from Cura (3D printing slicing software by Ultimaker), and the length of the square's side is calculated. The line distance is extracted for every 5% increase of the infill density. Finally, the heat flow by conduction, convection and radiation in the air cavity were added up. The resulting interpolation schemes can be found on Figure 4.13.

Results show that the heat flow is largely affected by the size of the cavity. The value of the equivalent thermal conductivity for lower densities has increased significantly. This is due to the fact that low density elements can create larger voids, where thermal convection reduces the thermal insulation quality. Additionally, three different results can be seen. These show the effect of the different nozzle sizes. Assuming the print width is equal to the nozzle width, the size of the cavity becomes larger when a larger nozzle is used. Consequently, a small nozzle will give better thermal properties as it can make smaller voids for the same density distribution. It can be noted that this convection problem can be avoided by filling the voids with additional insulation material. Although this might be achieved in certain cases, it brings difficulties to the manufacturing process, and is hard to achieve for large-scale components.



Figure 4.13. Adjusted material interpolation scheme for the thermal optimization problem, considering thermal convection and radiation in the cavities, versus the RAMP approximation of the numerical homogenization study.

To transform the results to the optimization process, a mathematical function is mapped onto these results. The resulting interpolation schemes are all based on the same basic function, but with different parameters for A, B, and C. The element thermal conductivity is presented as follows:

$$\lambda_{e}(x_{e}) = \lambda_{\min} + \left[A x_{e}^{2} + B x_{e} + C x_{e}^{-1}\right] (\lambda^{0} - \lambda_{\min})$$
w: $\lambda_{\min} = 0.025$ and $\lambda^{0} = 0.275$

$$(4.6)$$

The specific input for the parameters A, B, and C can be found in Table 4.2 and is constructed using the method of least squares. No physical experiments regarding the thermal properties were performed for this study.

Nozzle size (mm)	А	В	С
0.8	0.628	0.328	0.024
2	0.787	0.136	0.078
4	0.826	0.012	0.161

Table 4.2. Input parameter set for the thermal interpolation model.

Finally, it is worth mentioning that no results are presented for densities lower than 0.05, because the manufacturing of such low densities is deemed unrealistic, even for large-scale purposes. In the limit, when x_e goes to 0, the equivalent thermal conductivity depends on the size of the void that is being created. Although this value can be determined using boundary tracing techniques [11], this concept is not applied in this study. As such, the maximum value for the thermal conductivity was 0.42 W/mK, and the lower variable bound is adjusted.

4.4 Case study: 3D-printable roof structure

4.4.1 Mathematical formulation

In this case study, both the mechanical and thermal performances of a building roof component are studied. The goal is to improve the insulation quality of the roof structure, while still retaining adequate stiffness. A measure for the thermal performance is the thermal transmittance and is equal to the rate of heat transfer (in Watts) through one square meter of the structure, divided by the difference in temperature across the structure. As a result, the lower this value is, the better its insulation quality is. As a measure for the thermal transmittance, the thermal compliance is optimized. As for the mechanical performances, the idea is to minimize deflection to prevent problems with serviceability. This deflection is the degree to which a structural element is displaced under a certain load and can be linked to the structure's global stiffness and its structural

compliance. To combine these two opposing objectives, a weighted-sum multi-objective is created with weighting factors that can give more importance to one or the other. However, because the nature and magnitude of these objective functions are not known, additional scaling parameters are used to normalize the original values (i.e. a scaling operation is performed to make both compliances dimensionless). Finally, the maximum volume fraction, which equals to the amount of printing material that can be used, is being constrained and leads to the following multi-physics problem formulation:

$$\min_{x} \qquad w_{1} \frac{C_{s}(\mathbf{x})}{f_{1}} - w_{2} \frac{C_{t}(\mathbf{x})}{f_{2}}$$
s.t.
$$V(\mathbf{x})/V_{max} \leq 1 \qquad (4.7)$$

$$C_{s}(\mathbf{x}) = \mathbf{U}^{\mathrm{T}}\mathbf{K}_{s}\mathbf{U} = \sum_{e=1}^{N} E_{e}(x_{e})\mathbf{u}_{e}^{\mathrm{T}}\mathbf{k}_{s,e}^{0}\mathbf{u}_{e}$$

$$C_{i}(\mathbf{x}) = \mathbf{T}^{\mathrm{T}}\mathbf{K}_{i}\mathbf{T} = \sum_{e=1}^{N} \lambda_{e}(x_{e})\boldsymbol{\theta}_{e}^{\mathrm{T}}\mathbf{k}_{i,e}^{0}\boldsymbol{\theta}_{e}$$

$$0 \leq \mathbf{x} \leq 1$$

In this formulation, w_1 and f_1 are the weighing and scaling factors for the structural compliance $C_s(\mathbf{x})$, and w_2 and f_2 are the parameters that allow manipulation of the thermal compliance $C_t(\mathbf{x})$. $V(\mathbf{x})$ is the material volume, that linearly depends on \mathbf{x} , while V_{max} represents the maximum volume fraction (set to 50%). \mathbf{u}_e [8 × 1] and $\mathbf{\theta}_e$ [4 × 1] are the element displacement and element temperature vectors. Likewise, \mathbf{k}^{0}_{se} and \mathbf{k}^{0}_{te} stand for the element stiffness and element thermal conductivity matrices for an element with unit Young's modulus (E^{0}) and thermal conductivity (λ^{0}). The vector of design variables is again symbolized by \mathbf{x} , and N is the number of elements used to discretize the design domain.

4.4.2 Study parameters

The problem that is studied here, is the design of a fictitious roof structure for a – to be 3D-printed – polymer-based pavilion. This roof structure is shaped by several 6-meter-long elements, supported at the ends, and loaded by a uniformly distributed load (top and bottom surfaces). The boundary conditions and mechanical loads are presented in Figure 4.14. As can be observed, only half of the domain is modeled due to symmetry. The domain is discretized using a structured grid of 600×120 square finite elements with a unit length of 5 mm. The thickness of the design domain is 100 mm. The magnitude of the external loads is q = 1.5 N/mm, acting in the -Y direction. This value was derived from a 300 kg/m² plane load, being distributed over both the top and bottom surfaces. Additionally, also thermal boundary conditions are applied to the top and bottom surfaces. The inner boundary has a temperature of 20°C, while the outer boundary has a temperature difference between the bottom (inner surface) and the top (outer surface) is thus 20 K. Furthermore, the filter radius r_{min} is set to 4.0 and the allowable volume fraction is 50%.



Figure 4.14. Setup for the topology optimization study of a single span simply supported beam, subjected to uniformly distributed loads and a temperature difference of 20K between the top and bottom surface. Only the right symmetric half is presented.

The results are presented using a grayscale (0-1) colormap. The voids are displayed in white, and the solid material is shown as black. The intermediate densities thus represent the infill pattern that should be used. The mid-beam (bottom) deflections and the thermal transmittance (equivalent U-value) are also calculated to provide information about of the structure's structural and thermal performance. For the calculation of the thermal transmittance, the exchange of thermal energy between the body's surface and its surroundings is also considered. The values for the (inner and outer) heat transfer coefficients are $h_i = 7.7 \text{ W/(m^2K)}$ and $h_o = 25 \text{ W/(m^2K)}$.

The first set of results of the topology optimization study is now presented (Figure 4.15). These results are created using the thermal interpolation function for a nozzle size of 2 mm. The solutions are spread across two groups. The first group presents the optimized distribution of material in function of only one of the two sub-objectives. The first solution (Figure 4.15a) solves for maximum stiffness, while the second (Figure 4.15b) solves for maximum thermal efficiency.

As can be observed, the optimized solution in Figure 4.15a does not include any intermediate densities. A very stiff frame-like structure is created with a maximum (midbeam) deflection of 16.4 mm. This results in an overly good deflection state of L/367, which is below the required limit: L/300. However, the U-value of the beam element is very high (0.43 W/m²K). In contrast, the solution presented in Figure 4.15b only contains one type of grayscale material. The reason for this is obvious: $x_e = 0.35$ offers the lowest value for the equivalent thermal conductivity in the case 'nozzle width = 2 mm' (Figure 4.13). Would this study be conducted for the other nozzle sizes, the results would also converge to element densities that have the lowest value for the thermal conductivity (Nozzle 4 mm: $x_e = 0.45$ and nozzle 0.8 mm: $x_e = 0.20$). Although the U-value is now very good (0.19 W/m²K), this comes at the cost of a largely reduced stiffness; the mid-beam deflection now reads 92.0 mm (L/65).



(a) δ_{max} = 16.4 mm (L/367) / U_{mean} = 0.43 W/m²K

(b) $\delta_{max} = 92.0 \text{ mm} (L/65) / U_{mean} = 0.19 \text{ W/m}^2\text{K}$

Figure 4.15. Results of the topology optimization study: (a) minimum structural compliance design and (b) maximum thermal compliance design.

The optimized results presented in Figure 4.15 are the most extreme solutions. They define the outer limits of the multi-physics (also multi-objective) optimization problem. The first solution gives the stiffest roof component, whereas the second provides the best thermal resistance. Increasing the thermal performance of the first solution will always decrease its stiffness, while increasing the stiffness of the second result will always decrease its thermal efficiency.

The goal of the subsequent weighted-sum multi-objective optimization studies is to limit this performance deterioration and demonstrate the benefits of a multi-physics topology optimization study. An optimal trade-off is sought, while varying the importance of each of the sub-objectives. w_1/f_1 was fixed to 3000 in all results, while w_2/f_2 was set to 30000, 20000, and 8000 respectively. Figure 4.16 presents the multi-physics optimization study with the weighted-sum objective.

4.4.3 Optimized results

Figure 4.16 shows the optimized solutions where both objectives are activated. The importance of the thermal objective is introduced, adding 0.8 mm to the beam's deflection in Figure 4.16a. A mixed material lay-out distribution can be observed, where not only solid and void regions are created, but also intermediate material-density regions are


(c) δ_{max} = 19.3 mm (L/311) / U_{mean} = 0.24 W/m²K

Figure 4.16. Results showing the pareto optimal solutions of the multi-physics (weightedsum multi-objective) topology optimization study.

present. Most notable is that by allowing this small reduction in stiffness, the U-value of this beam element has become much better. For a 5% increase in deflection, the U-value has improved by 37%. This nicely exposes the benefits of a multi-physics topology optimization study, where a much more beneficial design can be found by analyzing not only one, but multiple criteria at the same time. For the subsequent solutions, the importance of the thermal objective is gradually increased to further lower the U-value so that it meets the current minimal requirements according to existing engineering codes (equivalent U-value roof < $0.24 \text{ W/m}^2\text{K}$). The result presented in Figure 4.16c finally hits the soft spot; a U-value of $0.24 \text{ W/m}^2\text{K}$ is reached and the mid-beam deflection arrives just below the required limit state of L/300.

4.4.4 Physical interpretation

Finally, there is one task remaining: the transformation of the mathematically optimized solution into a 3D-printable structure with different types of infill densities. Several strategies can be employed. However, we present a simple approach of which the methodology can be found in Figure 4.17. The process that is used, is called: image posterization; which entails a conversion of the continuous gradation of tone to several

regions of fewer tones. In this study, this is accomplished by decreasing the image's apparent bit depth (Figure 4.18a) and carried out because the slicing algorithm does not function with a continuous color gradation at this moment. However, such algorithms are in active development [12].



Figure 4.17. Schematic overview of the methodology that is used to transform the mathematical solution into a structure with different types of infill densities. Post-processing: (a) image posterization and (b) infill pattern generation.

4.4.5 Extension

Originally, the concept of mapping infill to densities of the TO problem was inspired by experimenting with different infill patterns used in desktop 3D printing. Since air may become trapped in the internal voids, certain 3-dimensional patterns (such as cubic or tetrahedral patterns) largely increase the structure's thermal properties. The lower the block's density, the lower its thermal conductivity becomes. However, from a certain point, the voids become too large, and convection inside the voids decreases its thermal performance. However, the principle of mapping material properties to a range of physical feasible materials or microstructures can be useful in other 3D printing technologies as well. For example, in large-scale 3D concrete printing, commonly only one extrusion

nozzle is used to cast plain (or fiber-reinforced) concrete. When the technology would allow a second nozzle, an extra material (e.g. a thermally efficient substitute such as "foamcrete" [13]) could be added to the production process.

Another approach would be to actively change the rheology properties of the concrete mixture inside of the main extrusion nozzle (Figure 4.18b). This idea touches the concept of a functionally graded concrete beam studied by Herrmann and Sobek [14] and is supported by advancements in active rheology control [15]. Such approach could really open up a whole new range of design opportunities for which the continuous density distribution could serve as a direct input for the TO optimization process. The specific requirements of the mixture could aim for a better assessment of the weight savings in concrete elements. The concept of a functionally graded concrete girder with weight optimization for a uniformly distributed load can be seen in Figure 4.19.



Figure 4.18. a) Image posterization showing the conversion of the continuous gradation of tones to several regions of fewer tones, and b) Principle of functionally graded concrete showing different densities in one concrete element (adopted from [14]).



Figure 4.19. Functionally graded concrete girder with weight optimization for a uniformly distributed load (adopted from [14]).

4.5 Discussion

The goal of this chapter was to better tune multi-physics topology optimization with additive manufacturing processes and to stimulate topology-optimized design for 3D-printable building structure. A novel multi-physics interpolation model was proposed that could link 3D-printing technology to density-based topology optimization. The structural and thermal material properties of a triangular infill pattern were analyzed and coupled to the mathematical design of the interpolation functions. Taking into account, among other things, thermal convection in the air cavities, a realistic penalization scheme was created. Subsequently, using this novel interpolation scheme, a roof component was optimized, and a design was generated with improved thermal and stiffness properties. Finally, the optimization study was able to generate several sets of optimized topologies with different trade-offs between structural integrity and thermal efficiency. This was realized by implementing a weighted-sum multi-objective.

Future studies could analyze the influence of different infill patterns such as honeycomb structures or tri-hexagon patterns. Especially, the analysis of 3D-infill patterns such as cubic, tetrahedral, or gyroid patterns could proof to offer valuable benefits for multi-physics topology optimization. Furthermore, this could strengthen the already existing and strong connection between topology-optimized design and additive manufacturing.

Additionally, 3D concrete printing could offer solutions to enable a shift from "stepwise gradation" to a virtually seamless gradation pattern, and would enable the production of complex geometries, where relatively few adjustments must be made to comply with the manufacturing constraints. Therefore, multi-physics topology optimization looks very promising and could potentially revolutionize certain design methods for building engineering, especially in a world where digital design meets 3D-printing technology.

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CHAPTER V

TOPOLOGY-OPTIMIZED CONCRETE GIRDER

This chapter is partially adopted from the following publications. The copyright of the original publications is held by the respective copyright holders, see the following copyright notices.

1. G. Vantyghem, V. Boel, W. De Corte, and M. Steeman, "Compliance, stress-based and multi-physics topology optimization for 3D-printed concrete structures," in First RILEM International Conference on Concrete and Digital Fabrication - Digital Concrete 2018, ETH Zurich, Switzerland, 2019, vol. 19, pp. 323–332.

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2. G. Vantyghem, W. De Corte, E. Shakour, and O. Amir, "3D printing of a post-tensioned concrete girder designed by topology optimization," Automation in Construction, vol. 112, 2020.

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5.1 Introduction

In this chapter, the link between topology optimization and 3D concrete printing is further elaborated on. The chapter starts with a general discussion on how TO can influence concrete structural design. For this, two conceptual applications are presented that demonstrate how the results from a TO study might directly influence the design and manufacturing process of a concrete component.

The first example discusses the use of classical structural TO as a method to directly extract an optimized printing pattern and/or printing path. It incorporates previously discussed TO methods and their results, and links them with the technology of 3D concrete printing. The second example considers how stress-based TO can enhance the topology of the concrete structural element by introducing stress constraints and by taking into account the differences between the compressive and tensile strength of conventional concrete. A case study example presents an optimized concrete specimen that considers the strength asymmetry that is associated with concrete. Also, some limitations and challenges are revealed.

Chapter 5 finally ends with the presentation of a topology-optimized concrete girder (Section 5.3) (Figure 5.1). The digital design and manufacturing of this concrete girder were carried out during the fourth year of this PhD study and it is fair to say that its realization made a relatively large impact in the field of 3D concrete printing (3DCP) ("In January/February 2021, thanks to the amount of citations, the paper was ranked in the top 1% of the academic field of Engineering" – WoS). The realized concrete girder resembles a footbridge (at lab scale) and was the first physical demonstration on how topological design in combination with 3D concrete extrusion printing allows for the creation of efficient concrete structures with reduced use of materials. The project brought together two emerging technologies that show great potential for realizing highly efficient concrete structures: topology optimization (TO) for simulation-driven design and 3DCP for the manufacturing of the resulting optimized shapes.

This section includes all steps from the conceptual design phase, the manufacturing challenges (including printing setup, assembly and integration of reinforcement), to the verification of the structural performances. For the latter, digital image correlation techniques were used, and a comparison was made to the numerical model. Additionally, some recommendations on the opportunities and limitations are given.



Figure 5.1. Topology-optimized concrete girder.

5.2 Topology optimization as a design tool for concrete structural elements

As discussed before, topology optimization has gained much popularity as a computational design tool for weight reduction of structural parts in automotive and aerospace applications [1] but has had only minor impact on the construction industry so far. Nevertheless, the negative environmental impact of concrete production clearly motivates the use of topology optimization for reducing material consumption in construction.

Topology optimization for concrete is not new. Over the past decades, it was suggested as a method for generating optimal strut-and-tie models [2-5]; for distributing materials based on the different strengths in tension and compression [6-8]; and for simultaneous

optimization of concrete and rebars [9–12]. In a recent review paper in Cement and Concrete Research [13], a slab element was topologically optimized for a uniformly distributed load, and a formwork was 3D printed using particle bed fusion and subsequently cast with UHPFRC. In another case, the same research group performed a topological optimization study for a concrete canoe competition, also using a 3D printed formwork cast with UHPFRC [14].

In the following subsections, the influence of general compliance-based TO on the design of concrete elements is briefly examined, where after the benefits of stress-based TO are explained.

5.2.1 Compliance-based topology optimization

At the start of this PhD project, TO algorithms were mainly being used for the automatic generation of strut-and-tie models for reinforced concrete design [2-5]. Strut-and-tie models can help with the dimensioning of steel reinforcements in so-called D-regions where Bernoulli theory is no longer appropriate. Strut-and-tie are designed to reduce load deformation response and in 1980, Schlaich stated that: "the stiffest truss model is the one that will produce the safest load-deformation response, because limiting truss deflection prevents large plastic deformations in the concrete. However, the engineering judgment required to obtain an accurate truss model was viewed as a drawback of the design approach" (citation from [4]). Now, maximizing stiffness correlates mathematically to minimizing the reinforcing steel's elastic strain energy. In other word, performing a minimum compliance TO design was suggested to provide excellent strut-and-tie models. Experiments and nonlinear finite element modelling confirmed this benefit [4]. Nevertheless, and although better performing, the increase in complexity of the models and reinforcement layout remained a draw-back of the method.

In contrast to using TO to find optimized strut-and-tie models, also the general shape of concrete elements can be optimized with it. To start, the TO results (or the optimized strutand-tie model) can serve as a direct input for the 3D printing path. Benefits can be

expected because the generated topology and the extracted print paths are in line with the principal elastic stress trajectories. Additionally, the complex pattern that is obtained from a TO study can also function as an internal skeleton. This skeleton can be produced from ultra-high-performance concrete (UHPC) [14], where the remainder of the design domain can be filled with a lower grade material. This material consisting of lower quality concrete would then make the whole design more cost efficient. Figure 5.2a illustrates such a conceptual 3DCP setup, and in Figure 5.2b a TO result is used to generate a strut-and-tie model or an optimized printing path.



Figure 5.2. Conceptual illustration of (a) 6-axis robotic arm used as a 3D concrete printer and (b) strut-and-tie model and printing path extracted from the TO result.

5.2.2 Stress-based topology optimization

A second potential benefit can be found when looking into stress-based topology optimization. As it is well known, concrete has a much higher compressive strength compared to its tensile strength. This results in a strength asymmetry that weakens the result of a traditionally compliance-based optimized design. Introducing a steel wire in the printing extrusion process [15] or having a fibre-reinforced material [16] can improve the tensile strength of the material, but it cannot fully eliminate this strength asymmetry. In

essence, the tensile and flexural tensile strength of the material will always be lower than the compressive strength.

The topology-optimized structures are thus oversized in the compression zones and would crack or fail rather easily in the regions with high tensile stresses. By introducing stress constraints in the topology optimization algorithm [17-19], optimized shapes can be generated which are optimized with respect to this strength asymmetry.

A first example is presented in Figure 5.3. Here, two results are shown that use stress constraints in the TO formulation. The result in Figure 5.3a shows the optimized material distribution for a material with equal compressive and tensile stresses. The result in Figure 5.3b gives the optimized topology for a compressive/tensile stress ratio of 2 (this moderate value for the compressive/tensile stress ratio is used for demonstration purposes only; true ratios for concrete would be much higher). It can be clearly observed that the bottom region of the second result is much wider than that of the first. The rationale behind this is that by having an increased amount of material at the bottom flange, more tensile action is allowed in this region. Finally, like the previous study, an improved concrete shape can be established by extracting the printing path from this optimized topology (Figure 5.3c).

5.2.3 Case study: optimized unreinforced concrete beam

To put some of these concepts into practice, an additional case study is presented. In function of a student competition in 2018, a small concrete specimen (span: 30 cm) was optimized using such stress-based TO approach. The 10th anniversary of the competition was celebrated with the theme "10". The goal of the competition therefore consisted of making the perfect concrete beam with a weight of 10 N and a failure load in bending of 10 kN. The test pieces were weighed on the day of competition and experimentally verified using a three-point bending test.

For the TO implementation, the Bresler-Pister criterium [20] was used as this yield surface presented superior results for high compressive/tensile stress ratios compared to a

Drucker-Prager yield criterion (Figure 5.4a). The maximum compressive strength was set to 60 MPa and the maximum tensile strength was set to 15 MPa. These values were based



Figure 5.3. TO with stress constraints using the Bresler-Pister criterium. (a) $\sigma_c / \sigma_t = 1$, (b) $\sigma_c / \sigma_t = 2$, and (c) showing the extracted printing path.

on real properties of an available mix. The mathematical formulation was adopted from [17] but replaced the Drucker-Prager yield surface with the Bresler-Pister formulation [20]. The design domain was optimized in 2D using the globally convergent version of MMA (GCMMA) (Section 2.1.2). In post-production, the resulting topology was translated into a 3D design with the help of Fusion 360 and using Abaqus simulations. A few manual design iterations (investigating the influence of slight design modification) were performed to further improve the design (for example, the rounding radius of hard edges of the 3D design).

Compared to a traditional rectangular beam, the optimized specimen is expected (based on numerical calculations) to outperform the rectangular design by a factor of 2 in terms of strength versus weight (for the same weight of both specimens, a doubling of the failure load in bending is observed). Additional gains (up to a factor of 4) were predicted by using UHPC combined with the optimized topology. The resulting shape can be observed in Figure 5.4b and the experimental setup on the day of the competition is presented in Figure 5.4c. For one of the beams that was tested, the highest score amongst all groups was achieved. This specific beam had a weight of 9.8 N and a failure load in bending of 10.3 kN.

Although this small beam was not produced using 3D concrete printing technology. It is feasible that a larger-scale beam would be, as no other conventional methods would be able to produce such complex formwork in any economical way.



Figure 5.4. (a) Mohr–Coulomb (red) and Drucker-Prager (blue) yield criterion (source: [17]), (b) optimized concrete specimen using stress-based TO, and (c) an experimental 3-point-bending test of the optimized specimen.

5.2.4 Discussion

Although compliance and stress-based topology optimization have shown to provide respectable insight when it comes to the design of concrete elements, its main application can be seen as a design tool for unreinforced concrete structures. For large functional structural concrete elements, such approach is not yet practical. Mainly, the lack of embedded tensile reinforcement in the optimization procedure hampers the applicability

of the methodology. In the case of the stress-based TO approach, the idea of using more material in the tensile zones also does not seem very efficient in the case of concrete structures, unless e.g. UHPC is used. A viable answer to this problem is presented in the next section.

5.3 Topology-optimized 3D-printed concrete girder

This section presents the design and manufacturing process of the topology-optimized concrete girder from Figure 5.1. This section combines many different aspects such as topology optimization, 3D concrete printing, and post-tensioning of concrete structures. In this introduction, the essential background is provided.

5.3.1 Introduction

The manufacturing of the topology-optimized girder is performed by 3D concrete printing. 3D concrete printing is a special type of additive manufacturing that has become very popular throughout recent years. It is a new tool in the toolbox of architects and construction companies and offers a quick and cost-efficient way of building large-scale engineering structures [21-23]. As defined by Buswell et al. [24]: "3D concrete printing (3DCP) works by precisely placing, or solidifying, specific volumes of material in sequential layers by a computer-controlled positioning process." Autonomous or semiautonomous 3D printers require minimal human surveillance, as such this could answer to the growing shortages of skilled workers [25]. Another outcome of the technique is that it disposes the need for conventional molding and allows for the creation of unique and complex shapes that were unattainable through conventional fabrication. By reducing the cost associated with nonstandard shapes, 3DCP gives virtual free rein to architects, designers and structural engineers enabling non-traditional design methods such as topology optimization [26-28].

However, as with any new technology, it also presents new challenges and complications [29]. A structural element that is well-designed according to traditional production methods and current standards may behave unexpectedly or even suffer damage as a result of the production process, i.e. 3DCP. The reason is mainly that the effect of the printing process itself on the final structural response as well as the effective macroscopic material properties are not fully explained today [24,30]. Consequently, the result from a topology optimization study might not perform as expected and fail to stand the test of time.

An additional and quite significant challenge is the problem of providing tensile resistance to 3D printed concrete. Currently, there is a big knowledge-gap on how to properly reinforce 3D-printed elements, and this is potentially the reason why the recent project (illustrated in Figure 5.5) received such great interest in the field of 3DCP.



Figure 5.5. Striatus 3D-printed concrete bridge before installment of the wooden finishing stairs. Image courtesy of Zaha Hadid Architects – Photograph by Tom van Mele [31].

Striatus is an arched segmented footbridge composed of 3D-printed concrete blocks [31]. The entire structure is held together through compression, needing no steel reinforcement and assembled without mortar. The 16×12 -meter footbridge is the first of its kind,

combing traditional techniques of master builders with advanced computational design, engineering, and robotic manufacturing [31]. Furthermore, the structure can be dismantled and reassembled in a different location.

Still, these kinds of structures (compression-only) also have their drawbacks. Especially, the risk of buckling under certain loads cannot be neglected. This is a major risk for all masonry structures, as they have little or no bending capacity other than that provided by the compression thrust.

Nevertheless, other methods that aim to provide tensile resistance in 3DCP structures were studied by many. The simplest method used today is insertion of steel rebars and grouting operations (Figure 5.6a). A more advanced study was performed by [32] where a modular approach for steel reinforcing of 3D printed concrete is proposed using subsequent insertion of flexural steel and joining the various modules using high-strength epoxy resins. In [33], a new in-process method is presented that embeds mesh reinforcement at the same time of printing the concrete layers. The reinforcement is placed during the placement of the cementitious layers as overlapping strips (Figure 5.6b). Alternatively, [34] also presented an effective in-process reinforcing technique by penetrating reinforcing bars through a predefined number of freshly printed layers to increase inter-layer bonding (Figure 5.6c), and in [35], the printing of concrete reinforced with steel fibers with different lengths and at different fiber volume contents are investigated. The study shows that more than 90% of extruded fibers align within 0°-30° from the filament orientation.

Finally, the concept that has received the most attention and has seen large-scale applications so far is the post-tensioning of 3D printed concrete [36,37] (Figure 5.6d). The advantage of this approach is that the printed concrete is stressed to a level so that only compression remains.



Figure 5.6. Providing tensile resistance to 3D-printed concrete by a) inserting traditional reinforcement cage [32], b) mesh reinforcements [33], c) penetrating reinforcing bars [34], and d) post-tensioning [36].

In the study that follows, the use of post-tensioning is focused on, with its geometry optimized simultaneously with the concrete distribution, thus alleviating the difficulty of introducing steel rebars or fibers in the 3D-printed concrete.

The starting point for this study, are the conceptual designs as obtained by Amir and Shakour [38]. We again rely on the density-based TO approach, where the structural topology is represented by a collection of density values (x) that can vary between 0 (void) and 1 (material) at discrete points in the design domain. However, for this design, a new procedure was proposed for concurrent optimization of the concrete layout and also the shape of the post-tensioning tendon embedded into it. The theoretical background is mostly discussed in [38]. Section 5.3.2.1 explains the mathematical formulation.

The following sections will subsequently focus on: (i) the design process, including the topology optimization approach, the post-processing methodology, and the print path generation; (ii) the manufacturing phase, including the segmental fabrication by 3D printing, the casting of the end blocks, and the assembly and post-tensioning of the girder; and finally (iii) the experimental testing and discussion section.

5.3.2 Design process

5.3.2.1 Topology optimization of the girder

As mentioned before, the design of the prestressed 3D-printed concrete girder is based on computational results from Amir and Shakour [38]. As the design procedure has been developed so far in 2D only and without precise design parameters, some adaptation of the design for the actual three-dimensional setting and material properties is necessary. Hence, the results from Amir and Shakour serve herein as a conceptual material distribution and tendon geometry, whereas actual dimensioning is performed in a postprocessing stage - described in the next section.

The particular design and manufacturing test case is that of a simply supported beam subjected to a uniform load. For topology optimization, we use a rectangular design domain with a length-to-height ratio of 10:1 that is discretized into a grid of 300×30 square finite elements, each element is associated with a density design variable. The setup of the problem is presented in Figure 5.7. Note that we utilize symmetry, so only one half of the design domain is simulated and optimized. At the beam's end, horizontal sliding supports are available through the complete height of the domain, so that the optimization procedure finds the best location of the supports that will meet the tendon's anchor.



Figure 5.7. Setup for topology optimization of a single-span beam subjected to a uniform load, utilizing symmetry. Gray represents the initial density value of 0.5 throughout the domain, while the density of the top surface is fixed to 1 (black) so that the loading area is not disturbed. The blue line represents the initial tendon shape.

The optimization seeks a design that minimizes the displacements at the top surface of the beam, due to the combined action of the external loads and the post-tensioning tendon. In mathematical terms, this design goal reads:

$$\min_{[\mathbf{x},\mathbf{P}]} \phi = (\mathbf{f}_{ext}^T \mathbf{u}_{total})^2$$

s.t.: $g = \frac{\sum_{e=1}^{N_E} x_e v_e}{\sum_{e=1}^{N_E} v_e} - V^* \le 0$
 $0 \le x_e \le 1, \qquad e = 1, ..., N_E$
 $\underline{\mathbf{P}} \le \mathbf{P} \le \overline{\mathbf{P}}$
 $\mathbf{K}\mathbf{u}_{total} = \mathbf{f}_{ext} + \mathbf{f}_{pre}$ (5.1)

where **x** is the vector of density variables; **P** the vector of tendon coordinates; \mathbf{f}_{ext} the vector of external forces; \mathbf{u}_{total} the vector of total displacements, $\mathbf{u}_{total} = \mathbf{u}_{ext} + \mathbf{u}_{pre}$; N_E the number of finite elements and density variables; v_e the volume of the *e*-th finite element; V^* the available volume fraction for the concrete; **P** and **P** the lower and upper bounds for the tendon coordinate movements; **K** the stiffness matrix of the concrete domain, related to the mathematical variables ρ and **P** via filtering and projection operations; and \mathbf{f}_{pre} the vector of prestress forces, that depends on P. We note that the square of the work ϕ is used in the objective so to avoid the erroneous solution that magnifies the negative work of \mathbf{f}_{ext} upon \mathbf{u}_{pre} .

For a standard beam with uniform cross-section, the required tendon force can be estimated as: $T_{\text{STD}} = \frac{M_{\text{total}}}{e_T + \frac{h}{6}}$, where M_{total} is the total bending moment at the critical

point (mid-span in the particular case); e_7 the maximum eccentricity of the tendon; and h the beam height. The design chosen for manufacturing and testing is the one obtained with $T = 0.8 \times T_{\text{STD}}$ and a concrete volume equal to 50% of the rectangular domain. The optimized concrete distribution and tendon layout are displayed in Figure 5.8.



Figure 5.8. Results of the topology optimization procedure for a single-span beam subjected to a uniform load. Black represents concrete; white represents void; cyan represents the tendon.

It should be emphasized that topology optimization of structures is a well-established methodology and its development towards 3D concrete printing applications can benefit from a vast body of knowledge. The particular case of optimizing the tendon geometry is quite new but results of several cases, including statically indeterminate beams, are available in the referenced article [38]. Moreover, ongoing work focuses on a more practical formulation that includes the tendon force as a design variable; curvature constraints on the tendon; and stress constraints in the concrete.

As topology optimization typically generates complex geometries, its coupling to additive manufacturing is natural and has been the focus of extensive research recently (see a recent review by Liu et al. [39] and references therein). The main challenge is to embed the printing limitations and constraints into the optimization formulation. So far, overhang limitations have been receiving attention, primarily by using projection or filtering operations [40,41]. As one of the limitations in concrete printing is the capacity to sustain self-weight during printing, formulations that embed gravity loads can be adapted to the particular case of concrete [42,43].

5.3.2.2 Design post-processing

The topology-optimized result from Figure 5.8 was adopted and transformed into 3D using Fusion 360 – Autodesk. The idea was to have a circular lower chord, so that the post-tension cable was evenly surrounded by a reasonable amount of concrete cover, thus avoiding local failure. Secondly, the upper chord of the girder was widened to allow for humans crossing the girder. We refer to this part as the deck side, as we purposely

envisioned a small bridge. Attention was paid not to make the deck too wide, since this would introduce unsafe transverse tensile forces in the top fibers of the upper chord. Taking these matters into account, a dynamic shape was finally created, see Figure 5.9. As can be observed, the 3D printing process was only used for the manufacturing of the contour shape of the girder; thereafter, the 3D-printed parts were assembled between two prefabricated end blocks, and the inner cavity was injected with a grout material. The function of the end blocks is two-fold. Primarily, they anchor the transverse bursting forces introduced by the post-tension force, and secondly, they are used to keep the separate parts together during the assembly and grouting process. Finally, as the project was meant to serve as a proof-of-concept, the girder was designed with a limited span width of 4.0 m, and the post-tensioning cable comprised a single 7-wire strand.



Figure 5.9. Computer-generated images of the to-be printed design with and without end blocks.

The analysis of the 3D-printed girder cannot be based on the 2D calculation. Since the optimization analysis is purely 2D, its stress results are not valid. Therefore, the model was studied using 3D finite element analysis, and the structural response under different load conditions was analyzed. The considered load cases were self-weight (g), self-weight + post-tensioning force $(g + f_{pre})$, and self-weight + post-tensioning force + live load (g+ $f_{\rm pre}$ + $f_{\rm ext}$). For symmetry reasons, only half of the girder was modelled. The domain was discretized using C3D4 and C3D6 elements (4-node linear tetrahedrons and 6-node linear triangular prisms) and comprises 125,555 elements. The use of complex quadratic elements was not deemed critical at this stage since the differences in results are likely neglectable for design purposes. Also included in the FE study are the post-tensioning strand in the lower chord and the end blocks. These were modelled using linear elements and connected using tie constraints. A diameter of 8.2 mm was used for the strand to generate an equivalent area of 52 mm² corresponding to 7-wire strand with a 9.3 mm nominal diameter, and surface-to-surface contact properties were defined to model the unbonded interaction with the concrete girder. The sliding formulation was defined as small with hard normal contact properties, and frictionless tangential behavior [44]. The material properties used in the FE-study can be found in Table 5.1 and the characteristics of the post-tensioning strand in Table 5.2. For clarification, the shape of the end blocks was not optimized.

Material	Young's modulus (MPa)	Poisson's ratio (-)	Density (kg/m³)
Concrete (C30/37)	32,800	0.2	2500
Steel	190,000	0.3	7800

Table 5.1. Material properties as used in the FE-model.

Туре	3/8"
Diameter (mm)	9.3
A _p (mm²)	52
f _{pk} (N/mm²)	1860
Max. prestress force (kN)	77.4

Table 5.2. Properties of the post-tensioning 7-wire strand

It should be noted that for the numerical analysis, the girder (consisting of an outer layer of 3D-printed concrete and filled with grout infill material) was modelled as a solid section with averaged properties. Therefore, the Young's modulus and its associated concrete grade had to be estimated based on the 'hybrid' nature of the structure. The ratio between the 3D-printed outer layers and the internal cavity volume was about ±85%, as such a mean value was calculated, taking into account that too much water was added to the grout premix. In the end, an equivalent concrete grade of C30/37 was selected for the analysis of the girder. Concerning the validation part, several other concrete grades were added and discussed. Based on this concrete grade with a characteristic cylinder compressive strength of 30 MPa (C30/37), the following design limits were extracted in accordance with EN 1992-1-1: the design compressive strength of the concrete was constrained to 20 MPa and the design tensile strength was limited to 1.35 MPa.

The results of a linear elastic analysis based on the aforementioned geometry, loads and material properties can be found in Figure 5.10. The first study (Figure 5.10a) only considers the self-weight of the girder. Results show that the maximum principal stresses in the girder are mostly below these ultimate stress states. The average tensile stress in the lower chord is only 1 MPa. The second result (Figure 5.10b) presents the girder when the live load is also applied. This load case (without the post-tensioning) will not occur in reality but is included to assess certain risks. The live load is set equal to 17 kN distributed

equally across the top surface (i.e., $f_{ext} = 10 \text{ kN/m}^2$). The tensile stress in the girder now exceeds the design tensile strength.

The third result (Figure 5.10c) considers the self-weight and the post-tensioning. Due to the prestress force, the lower chord of the girder is loaded in compression, while tensile forces arise in the top part of the girder. The prestress force was set at 50 kN. As can be observed, the occurring stresses are mostly within the design limits of the material; except for a peak stress concentration in one of the struts. The maximum compression stress in the lower chord is around 5 MPa, and the maximum tensile stress in the upper chord is 0.5 MPa. The final analysis (Figure 5.10d) shows the principal stresses of the girder when all loads are combined: the self-weight of the girder plus the prestressing and external forces. As expected, in prestressed concrete design, the girder appears to be in a neutral state, where most of the tensile (and compressive) stresses are eliminated. A small peak of tensile stresses does remain at the bottom.

Furthermore, in bridge design engineering, asymmetric design loads and concentrated convoy loads should be considered as well. A topology optimization study concerning such additional load cases was performed by Jansseune & De Corte [45]. However, as the approach was not yet combined with the algorithm described in Section 5.3.2.1, asymmetric design loads and concentrated convoy loads were not considered. Additionally, the stress distribution in the end blocks was removed from the results as they did not accurately predict the internal stresses. The reason for this is the inaccurate modeling of the anchorage zones and the simplifications in the material model. Advanced FE models such as proposed by Van Meirvenne et al. [46] could provide more insight. However, their incorporation is beyond the scope of the current study. The amount of rebar in the end blocks was therefore calculated manually, and in accordance with LRFD Bridge Design Specifications – AASHTO [47].

Finally, some additional safety measures were taken in the final design. Additional rebars were added to the structure to protect it from unforeseen stress concentrations considering the segmentation in the printing process (see further). Therefore, in every diagonal strut, two rebars of Ø12 were added (Figure 5.10e).

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Figure 5.10. Principal stress plots for the different load cases. (a) self-weight, (b) self-weight + live load, (c) self-weight + prestress force, (d) self-weight + prestress force + live load, and (e) dimensioning of the reinforcements.

5.3.2.3 Print-path generation

As expected, the girder could not be 3D printed in one piece. As such, a similar approach as observed in the production of the 3D-printed bicycle bridge in Gemert, The Netherlands [36] was adopted. In that project, six identical pieces were individually printed, then rotated on their sides and post-tensioned towards one another. Similar to that approach, this topology-optimized design was subdivided into several parts (Figure 5.11a) and for each of them the contour shape was 3D printed. Since each element had a maximum height of 400 mm and a maximum total weight of 30 kg, they could be printed with a minimal amount of support material, where taller prints would collapse under self-weight load. The subdivision of the 3D design was performed in Rhinoceros using the Grasshopper plugin. A custom Grasshopper script was created to slice the volumes and to generate the RAPID code with robot control instructions (Figure 5.11b). Considering the symmetry of the girder, nine RAPID files were created, each containing roughly 4000 lines of code needed to control the ABB robotic arm.



Figure 5.11. Generation of the printing path by visual scripting techniques in Rhinoceros & Grasshopper showing (a) all the different parts and (b) the tool path for part 8 and 9.

5.3.3 Manufacturing process

The manufacturing process of the girder was separated into three phases. Firstly, the actual 3D printing of the eighteen girder segments and the casting of the two end blocks is presented. Secondly, the assembly of the different parts is discussed, including the grouting process and the integration of rebars, and finally, the post-tensioning of the lower tendon.

5.3.3.1 Printing the segments

The 3D printing of the girder segments took place at the Magnel-Vandepitte Laboratory for concrete research and at Vertico [48]. Because both setups are very similar, only the printing setup of the former is described (Figure 5.12). The setup comprises: (i) a robotic arm, (ii) a mortar (screw) pump for cementitious material, and (iii) a concrete 3D printing mixture.

The robot is of the type: ABB IRB6650 – with a range of 3.2 m and payload of 125 kg – and has 6 degrees of freedom (DOFs), which enable printing in almost every orientation and tool alignment. It is one of two systems that is frequently used by researchers and companies in the field of 3D concrete printing. In this experiment, the robot actuator remained perpendicular to the horizontal plane. The pump has a delivery rate between 2 and 29 L/min and can handle pressures up to 30 bar. The mortar recipe was adopted from '*De Huizenprinters*' [49], and consists of 51.1% dried sand 0/2, 34.7% Portland cement CEM I 52.5 R, 13.2% water, and 1.0% water retention agent (these percentages are in mass, i.e. % of the total weight). The latter ensures that the water is more retained, and the mix gets its desired thixotropic behavior and prevents the occurrence of pressurized bleeding. The print nozzle has a Ø25 mm opening and is set to print at 80 mm/s. The total printing time of the segments was estimated to be around 24 hours. The actual printing was performed in three working days and required at least three operators simultaneously. The 3D-printed segments were allowed one night of settling, whereafter they could be removed from the printing bed and be transported.



Figure 5.12. 3D concrete printing set-up at the Magnel-Vandepitte Laboratory showing the realized post-tensioned concrete girder in the back.

5.3.3.2 Casting of the end blocks

Traditional casting was used for the realization of the end blocks. A wooden formwork was made and steel rebars and spirals were inserted (Figures 5.13a and 5.13b). In each end block, not one but three anchorage systems were built in to later install the post-tensioning strands. The main strand in the middle and two supplementary deck strands (also ø 9.2 mm) were provided for safety reasons (transport and inverse loading conditions during assembly). For this, steel ducts were embedded in the concrete to allow for the post-tensioning strands and their hoses to run through the concrete mass. In addition, perpendicularly to the main duct, a steel reinforcing plate was embedded to transfer the post-tensioning force from the wedge into the concrete end block (Figure 5.13c).

5.3.3.3 Assembly, integration of reinforcements & grouting

After the 3D printing of the girder, the next step was to assemble all parts, including adding the reinforcement bars, and to fill the inner cavity with a grout mortar. This grout was



Figure 5.13. Manufacturing of the end blocks: (a) CAD design – dimensions in [cm], (b) wooden formwork and added reinforcements, and (c) the resulting end block with a visual on the steel reinforcing plate.

injected after a sufficient hardening/settling period of the 3D-printed segments (i.e., several weeks).

First, the printed elements (Figure 5.14a) were positioned with their deck side flat on the ground. By positioning the parts this way, only a limited amount of support structure was required. Foam blocks were used to support the cylindrical sections in the mid part of the girder. The joining of the different parts was performed from one end to the other to enable the insertion of the steel rebar (Figure 5.14b). To keep them in place, traditional plastic spacers were used. The post-tensioning strands – surrounded by a plastic sheathing – were also brought in position. Attention was given to the positioning of the main strand, which had to run as centrically as possible to avoid secondary bending in the lower chord. To close the gaps between the printed elements, the two deck strands were prestressed using a very small force (5 kN). Next, the joints were sealed off with a PU foam gun to close any remaining gaps (Figure 5.14c). Finally, four 30 mm holes were drilled in the girder. The first and last hole were drilled 400 mm from the supports, and holes two and three were drilled 1000 mm from mid span. One of the center holes was used as inlet for the grout material, while the other holes served as an air outlet. The same pumping system as for the 3D printing was used to transport the grout. The grouting material was a high-

quality shrinkage compensating high-strength seal mortar [50] (compressive strength ~60 MPa; tensile strength ~12 MPa). However, since the consistency of the mortar had to be adjusted during pumping by adding water, the quality of the material is not certain, but likely somewhat lower. Whenever the mortar rose from one of the remaining holes, pumping was paused, and the hole was sealed. A small amount of mortar leaked through an unclosed gap in the mid-bottom section of the girder. Finally, the girder was allowed two weeks of hardening.



Figure 5.14. (a) 3D-printed segments, (b) positioning of reinforcement, and (c) assembly and grouting process.

5.3.3.4 Post-tensioning

After a hardening period of 14 days, the girder was lifted from the ground, and the main tendon was slightly tensioned (10 kN); next, the girder was flipped in its upright position. During this elevation and rotation process, the girder was supported by polyester lashing belts. The final structure was placed on supports with a rubber slab below each end block (Figure 5.15a) and a preliminary verification of the girder's strength was performed by carefully allowing people on the girder (estimated weight: 7 kN). Finally, the full posttension force of 50 kN was applied, and the upward deflection was measured using several

dial indicators and digital image processing (Figure 5.15b). The upwards deflection of the upper chord was around 10 mm, while the lower chord's maximum final deflection was around 18 mm. The (excessive) deflection of the lower chord (especially in the mid-span) was more than what the FE-model predicted and could be attributed to secondary bending of the lower chord (P- δ) and/or due to misalignment of the post-tensioning strand within. This issue is further discussed in the next section.



(b)

Figure 5.15. (a) Completed girder and (b) digital image correlation software capturing the deflection (in mm) of the manufactured girder during the post-tensioning phase.

5.3.4 Experimental testing

Finally, load/displacement tests were performed on the manufactured girder. The goal was to verify the service load performance of the optimized shape. Since the original objective of the optimization study was to seek a design that minimizes the displacements at the top surface of the beam, the focus of the experiment was also on the upper chord. Digital image processing [51] was used to measure the deflection of the girder, with the white markers being the area of interest. The displacements of these markers were tracked, and the measurements were verified using digital dial indicators (Figures 5.16 and 5.17). Both the upper and lower chord were registered. In the end, the experimental results were compared to the original FE analysis.



Figure 5.16. Experimental setup to monitor deformations.

In Figure 5.18, the numerical results and markers are presented using lines and points, respectively. The upper chord (Figure 5.18a) shows a good fit between the experimental and numerical results. Most of the tracked markers are within the grey area, which shows the numerical deflection values for different concrete grades. The small deviations from the numerical values can be attributed to the sectional assembly, and the use of linear



Figure 5.17. Registration of the displacements (in mm) of the concrete girder by digital image correlation (DIC).

elements in the FE analysis (these elements are known to be overly stiff hence will underestimate deflections). On the other hand, the lower chord (Figure 5.18b) shows a much higher deviation from the numerical result. The main deviation arises around the mid-section, indicating an effect of secondary bending in the lower chord (P- δ). This can be attributed to tolerances in the position of the post-tensioning strand within the lower chord, as well as to the sectional assembly. During the post-tensioning phase, the upwards deflection of the mid-span was already higher than what was predicted. Therefore, when loading the structure, the initial positions of the tracked markers in the upward deformation that is being relaxed, is adding up to the deflection of the girder. In larger structures, the positioning of the post-tension strand should be easier to control and eliminate this problem.





Figure 5.18. Registration of the displacements of the concrete girder compared to the numerical results for (a) the upper chord and (b) the lower chord.

5.3.5 Discussion

This section presented a complete digital design-to-manufacture process that combined topology optimization, 3D concrete printing and post-tensioning. As additive manufacturing in general offers relatively large design freedom, it can promote the reduction of material consumption when coupled with techniques such as topology optimization. The complex shapes that arise from topology optimization procedures challenged the manufacturing techniques and material limitations of 3D concrete printing – hence our main purpose was to demonstrate the feasibility of the process and to show a proof-of-concept in the form of a scaled girder. By careful segmentation of the design into printable parts, followed by joining them with post-tensioning tendons, we obtained a viable concrete girder that could sustain the loads for which it was designed. The use of post-tensioning, with its geometry optimized simultaneously with the concrete

distribution, alleviated the difficulty of introducing steel reinforcement in 3D-printed concrete.

Based on the performance of the optimized beam, in terms of mid-span deflections under self-weight (590 kg) and live loads (540 kg), we can predict material savings of roughly 20%. This estimate is based on finding a T-section girder with the same flange size (585 mm by 97 mm) and the same overall depth (388 mm) as the optimized beam, giving the same total deflection. The values for the defection (adopted from the numerical model) are 0.28 mm under self-weight and 0.24 mm under 540 kg. The resulting volume of the T-section concrete girder is nearly 20% higher than that of the optimized beam – hence clarifying the incentive to employ topology optimization and manufacture complex geometries. Furthermore, for larger bridge structures, where the self-weight component becomes even more significant, additional savings can be expected. More details of this comparison can be found in Appendix 3.

Concerning some of the current limitations, first of all, a clearly defined design protocol is missing. A mismatch is still present regarding the compatibility of the 2D topology optimization script and the development of the 3D design. The current approach was mainly based on engineering justice and was performed quite crude. Were this proposed methodology to gain traction in the construction industry, ideally the TO script should be developed to work in 3D as well. Additionally, as it was only optimized for a uniformly distributed load, the optimization only considered global deflection and stiffness, while in bridge design engineering, stresses and local deflection are of critical importance.

In addition, no standards (or international regulations) were yet applied on this case. As such the essential performance requirements to protect the citizens cannot be guaranteed at this moment. The need for providing an internationally harmonized structural design and material acceptance framework for DFC structures is further discussed in [52]. For example, in Europe this means that the safety in the construction industry needs to be ensured at three different stages: the building preparation or design (EN-1992-1-1), building construction (EN 13670), and materials supply (EN 206). However, the particularities of a 3DCP structure are not worked out in any of these regulations. It is
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clear that a considerable amount of research still needs to be performed to consider DFC as a standard construction method.

Furthermore, the manufacturing process described in this work does not yet reveal a clear economic benefit. Nevertheless, the focus was on the feasibility of the process, combining a computational-intensive design method and robotic 3D concrete printing. The realistic economic benefits of 3D printing over traditional manufacturing methods are still a matter of research, not only in the context of concrete but also in more established technologies such as powder-based metal printing. Therefore, we do not claim any immediate economical savings. However, as the technology improves and consolidates, the expected key benefits will be: i) Reduction of concrete consumption, due to optimization and realization of complex geometry that is not necessarily feasible in traditional manufacturing; ii) No need of molding as in concrete casting; and iii) Possibility of (autonomous) robotic manufacturing.

This study also opens up many research topics that require further investigations. At first, the deviations between the numerical model and the measured deflections at the lower chord need to be investigated. One possible explanation could be secondary bending moments due to imperfections in the alignment of the tendon. For this, it would have been interesting to assess the deflection of the lower chord, modelled by itself, assuming different eccentricities. Another factor of uncertainty is the determination of the equivalent material characteristics of the hybrid structure (3DCP segments and grout infill). From the computational design perspective, full 3D topology optimization with prestressing still needs to be addressed, while accounting for manufacturing limitations and uncertainties poses further challenges. From the manufacturing perspective, future work should focus on improving the buildability of printable construction materials, i.e. the ability of extruded material to retain its shape under sustained loads. This may be achieved in the future using active rheology (or stiffening) control as well as a setting-on-demand mix design approach. Progress in this field could increase the number of continuous layers that can be printed (or stacked) without failure and boost the maximum degree of overhang in a design.

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In the broader context of 3D printing shape-complex structures, a recent interesting case was also presented by Kinomura et al. [37], who manufactured a topology optimized pedestrian bridge consisting of 44 3D printed, prestressed concrete segments. A holistic verification study via continuing numerical investigations of the design is included in the publication. E.g., it is confirmed that any fracture occurrences are not observed even under maximum loading and the manufactured bridge behaves as an elastic unity.

Finally, having a finite element-based virtual simulation model for the 3D concrete printing process is valuable. Such model could predict values of strength, internal stresses, critical locations, and initial deformations during the printing progress. Based on the outcome from a FEM-based calculation, a particular design could be marked unfit for printing, providing additional insight on process parameters that could be tuned in order to increase the chance of success in printing the analyzed design; thus, avoiding costly physical experiments.

Finally, if all these challenges can be overcome, the author sees great potential in using this design methodology for the manufacturing of large-scale post-tensioned concrete bridges.

5.3.6 Acknowledgements

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- 3D modelling and FE-analysis: Research Group Schoonmeersen Department of Structural Engineering and Building Materials - Ghent University
- Rheology mix design and manufacturing process: Mortar recipe from wiki.bouwkoppel.nl | dehuizenprinters - Vertico
- Assembly and post-tensioning: Magnel-Vandepitte Laboratory Department of Structural Engineering and Building Materials - Ghent University

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CHAPTER VI

CONCLUSIONS

6.1 Conclusions

The goal of this doctoral thesis project was to enhance the design process of constructionrelated components by harnessing the power of generative design optimization strategies such as topology optimization. Topology optimization was described in the introduction (Chapter 1) as an innovative mathematical design technique that allows for the exploration of 'general' shapes with the goal of maximizing performances. The use of topology optimization as a design tool is different from the other design practices in the sense that the design can attain any shape within the predefined design domain. This stands in contrast with the general methodology where most of the time a specific design configuration is decided on by the designer(s) or engineering team. In Chapter 2, a comprehensive overview of classical (structural) topology optimization schemes was given and several important algorithms and methods were described. Also, some extensions were elaborated on, and various examples presented.

Of course, the idea for this research project was not only to study the state-of-the-art. The main objective was to explore the full potential of topology optimization for the design of construction-related components (within the building envelope). Especially, the suggestion/idea to optimize not only structural performances, but also certain multiphysics performance indicators had the main interest. Consequently, Chapter 3 dug deeper into this subject.

In this chapter, topology optimization was performed considering simultaneous structural and thermal analyses. The design of a thermally efficient masonry block and a brickwork support bracket were addressed. In both studies, the goal was to reduce localized thermal heat flow (cold bridging) and thermal resistance, while retaining sufficient structural stiffness and integrity. Several problems related to multi-physics topology optimization procedures were examined and the benefits of a two-material implementation were presented. The aim was to improve the robustness of such multi-physics optimization processes and to provide a methodology to accurately determine the beneficial design solutions.

In the first case study (thermally efficient masonry block), it was shown that topology optimization can generate a large diversity of different material distributions. It was demonstrated that the suggested methodology could inspire designers early on in the design process and help them to explore new design ideas by thinking outside the box. An advantage of the approach was that TO could aid engineers and designers to understand better the underlaying mathematical physical problem and aid them in the optimization of their designs.

In the second case study (brickwork support bracket), the preliminary results showed that a small admission on the material usage or allowable volume fraction could largely improve the thermal performance (and thus reduce cold bridging) while retaining (almost) equal stiffness. A small additional cost of raw materials therefore proved to be beneficial in this kind of design problem. However, further on, a first parametric study revealed that a combined structural and thermal topology optimization approach can cause serious confusion regarding material interpolation. The penalization parameters that are used to construct the interpolation curves for the density - thermal conductivity relation are hard to determine, and when choosing improbable parameters, interesting design solutions could go by unnoticed. To solve this problem, a multi-material topology optimization approach was included which added an extra set of design variables. In this way, a new type of material could be added with predefined properties. Using this multi-material, multi-physics approach, innovative and new topological solutions were found. Choosing the right materials remained important, but unlike the traditional approach, the design solutions were unambiguous, and a realistic optimum could be obtained. The final study then optimized a brickwork support bracket inspired by a real-world design of Halfen and an improved design proposal was found that included two ABS thermal breaks.

This first set of studies proved that finding optimal design solutions requires a broad knowledge in many different fields, as the design of construction-related components is unavoidably a multidisciplinary activity. Combining several disciplines within the topology optimization study should give much better (and faster) solutions in comparison to carrying out many individual (single-)disciplinary optimizations.

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With respect to this chapter, future research could focus on experimenting with more than one additional material and try to include a cost function to incorporate the price variance of different materials. Also, the material strength and the strength of the bonding between the different materials and the incorporation of nonlinear material models into the algorithm could become relevant. Additionally, as mentioned already, the design of construction-related components is affected by many different disciplines. In addition to structural and thermal performances, other aspects such as acoustics, fire safety, hygrothermal effects, or durability and sustainability could be added.

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A secondary objective in this research project was to consider the production method that would work best in harmony with the results from such a topology optimization study. As these highly optimized designs were often very complex in shape, additive manufacturing was identified as the most promising. Additionally, this idea of using additive manufacturing presented several opportunities, especially, the possibility to produce custom infill densities on the fly. This aspect was elaborated on in Chapter 4.

The goal of this chapter was to better tune multi-physics topology optimization for additive manufacturing processes and to stimulate topology-optimized design for 3D-printable building structures. A novel multi-physics interpolation model was proposed that linked 3D-printing technology to density-based topology optimization. The structural and thermal material properties of a triangular infill pattern were analyzed and coupled to the mathematical design of the interpolation functions. Taking into account, among other things, thermal convection in the air cavities, a realistic penalization scheme was created. Subsequently, using this novel interpolation scheme, a roof component was optimized, and a design was generated with improved thermal and stiffness properties. Finally, the optimization study was able to generate several sets of optimized topologies with different trade-offs between structural stiffness and thermal efficiency. This was realized by implementing a weighted-sum multi-objective. By studying the pareto frontier, the decision-making process of the designer can be boosted, as it can provide the designer with additional domain knowledge and aid in choosing the final solution.

Future studies could analyze the influence of different infill patterns such as honeycomb structures or tri-hexagon patterns. Especially, the analysis of 3D-infill patterns such as cubic, tetrahedral, or gyroid patterns could proof to offer valuable benefits for multi-physics topology optimization. Furthermore, this concept could strengthen the already existing and strong connection between topology-optimized design and additive manufacturing.

Additionally, it was proposed theoretically that 3D concrete printing could offer solutions to enable a shift from "stepwise gradation" to a virtually seamless gradation of internal densities. This would enable the production of complex geometries, where relatively few adjustments must be made to comply with the manufacturing constraints. Therefore, the topic of variable-density multi-physics topology optimization for 3D-printable building structures looks very promising and could potentially revolutionize certain design methods for building engineering, especially in a world where digital design meets 3D-printing technology.

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Thus, in the last chapter (Chapter 5), the aim was to link 3D concrete printing with topology optimized designs for the construction industry. As additive manufacturing in general offers relatively large design freedom, it can allow for the reduction of material consumption when coupled with design techniques such as topology optimization. The complex shapes that arose from topology optimization procedures challenged the manufacturing techniques and material limitations of 3D concrete printing. With this chapter, the opportunities for structural engineering of digitally fabricated concrete components were explored. A complete digital design-to-manufacture process was presented, combining topology optimization, 3D concrete printing and post-tensioning. The main focus was to demonstrate the feasibility of the process and to present a first proof-of-concept in the form of a 3D-printed scaled concrete girder (i.e. small bridge).

One of the first steps described the transitioning from a 2D topology optimization result to the 3D design with help from FEM analysis, followed by careful segmentation of the 3D

design into printable parts, the printing process and the joining of the different parts with post-tensioning strands. A viable concrete girder was obtained that could sustain the loads for which it was designed. The use of post-tensioning, with its geometry optimized simultaneously with the concrete distribution, alleviated the difficulty of introducing steel reinforcement in 3D-printed concrete.

Based on the performance of the optimized beam, in terms of mid-span deflections under self-weight and live loads, material savings of roughly 20% were achieved. The manufacturing process described in this work does not yet reveal a very clear economic benefit. However, the economical vision, that it could trigger a "chain of improvements" in the construction industry, is presumable.

The realistic economic benefits of 3D printing over traditional manufacturing methods are still a matter of research, not only in the context of concrete but also in more established technologies such as powder-based metal printing. Here, the focus was on the feasibility of the process, combining a computational-intensive design method with robotic 3D concrete printing. As the technology improves and consolidates, the expected key benefits will be: i) a reduction of concrete consumption, due to optimization and realization of complex geometry that is not necessarily feasible in traditional manufacturing; ii) no need of molding as in concrete casting; and iii) the possibility of autonomous robotic manufacturing on site.

6.2 Recommendations and outlook

Now, to finally return to the original title and perhaps the main question of this PhD project: "Is there potential in combining topology optimization and 3D printing in the construction industry?", I think the answer is clearly: Yes! Based on the findings of this PhD study, the future of combined design optimization and digital fabrication looks bright. Nevertheless, many important challenges remain. First of all, in many of the specific cases and algorithms, assumptions were introduced to simplify the problem. While often the

assumptions were accounted for, there is always the risk that the results are not valid in their application. Additionally, some extensions were discussed that could prove to be very beneficial if they were to be added. For example, the introduction of stress constraints in certain topology optimization studies, or the strength of bonding for multi-material design problems. Concerning the topology optimized girder, also a clearly defined design protocol was still missing. The current approach where the 2D topology optimized solution was transformed into a realistic 3D design, was mainly based on engineering justice and influenced by aesthetic expectations. This leaves much room for further improvements and follow-up research studies.

In my opinion, the main benefit (impact within the construction industry) will be found in single variable-based multi-material optimization for concrete (building) structures. For example, if the rheology of a concrete mixture could be changed actively inside the 3D printing extrusion nozzle, the topology optimization algorithm could determine the exact requirements of this mixture at every location, allowing for a precise assessment of the weight savings and performances. Being able to deposit a strong concrete mix where it needs to be strong, less strong (and less costly) where it is allowed, and perhaps thermally insulating where the design could benefit from it. This concept of having a functionally graded concrete available was only slightly touched in this work. Regrettably, the technology was not there yet to include some preliminary experimentation within this subject. I sincerely hope future studies could tackle this concept.

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OVERVIEW OF MY PHD YEARS AT GHENT UNIVERSITY

LIST OF PUBLICATIONS

JOURNAL PUBLICATIONS (A1)

- G. Vantyghem, V. Boel, M. Steeman, and W. De Corte, "Multi-material topology optimization involving simultaneous structural and thermal analyses," Structural and Multidisciplinary Optimization, vol. 59, no. 3, pp. 731–743, 2019.
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APPENDIX 1 SENSITIVITY ANALYSIS

Several distinct methods exist for the computation of gradients: **approximate, numerical methods**, or **exact, analytical methods**. For the first set of methods, the sensitivity of the function with respect to one variable of the design domain is approximated by giving the value of the variable a small perturbation. The FE analysis is reiterated with this value and the response is compared to that obtained for the nominal value. As the analysis needs to be repeated for each variable of the design domain, the method becomes very time consuming and computationally expensive for problems with many design variables. The method should only be considered if the functions that are used, are not available analytically.

In contrast, analytical methods can be used when the functions are differentiable. In structural optimization and topology optimization problems, analytical gradients are used almost exclusively. In the following paragraph, the derivation of the gradients for the compliance function is presented.

Let us again define compliance $C(\mathbf{x})$ as

$$C(\mathbf{x}) = \hat{C}(\mathbf{x}, \mathbf{U}(x)) = \mathbf{F}^{\mathrm{T}}\mathbf{U}.$$
 (A1.1)

Due to the nested formulation, the displacements **U** are implicitly function of **x**, meaning that the partial derivative of $C(\mathbf{x})$ conceptually reads as follows:

$$\frac{\partial C(\mathbf{x})}{\partial x_e} = \frac{\partial \hat{C}(\mathbf{x}, \mathbf{U}(\mathbf{x}))}{\partial x_e} + \frac{\partial \hat{C}(\mathbf{x}, \mathbf{U}(\mathbf{x}))}{\partial \mathbf{U}(\mathbf{x})} \frac{\partial \mathbf{U}(\mathbf{x})}{\partial x_e}.$$
(A1.2)

When differentiating the state equation ($\mathbf{F} = \mathbf{KU}$), the sensitivity of the displacements can be determined.

$$\frac{\partial \mathbf{F}}{\partial x_e} = \frac{\partial \mathbf{K}(\mathbf{x})}{\partial x_e} \mathbf{U}(\mathbf{x}) + \mathbf{K}(\mathbf{x}) \frac{\partial \mathbf{U}(\mathbf{x})}{\partial x_e},$$

which after rearrangement of terms read

$$\frac{\partial \mathbf{U}(\mathbf{x})}{\partial x_e} = \mathbf{K}^{-1}(\mathbf{x}) \left[\frac{\partial \mathbf{F}}{\partial x_e} - \frac{\partial \mathbf{K}(\mathbf{x})}{\partial x_e} \mathbf{U}(\mathbf{x}) \right].$$
(A1.3)

Inserting Eq.(A1.3) into Eq.(A1.2) yields

$$\frac{\partial C(\mathbf{x})}{\partial x_{e}} = \frac{\partial \hat{C}(\mathbf{x}, \mathbf{U}(\mathbf{x}))}{\partial x_{e}} \cdot + \frac{\partial \hat{C}(\mathbf{x}, \mathbf{U}(\mathbf{x}))}{\partial \mathbf{U}(\mathbf{x})} \mathbf{K}^{-1}(\mathbf{x}) \left[\frac{\partial \mathbf{F}}{\partial x_{e}} - \frac{\partial \mathbf{K}(\mathbf{x})}{\partial x_{e}} \mathbf{U}(\mathbf{x}) \right]$$
(A1.4)

Because from Eq. (A1.1)

$$\frac{\partial \hat{C}(\mathbf{x}, \mathbf{U}(\mathbf{x}))}{\partial \mathbf{U}(\mathbf{x})} = \mathbf{F}^{\mathrm{T}}, \qquad (A1.5)$$

and also from Eq. (A1.1)

$$\frac{\partial \hat{C}(\mathbf{x}, \mathbf{U}(\mathbf{x}))}{\partial x_e} = \frac{\partial \mathbf{F}}{\partial x_e} \mathbf{U}$$

Equation (A1.4) simplifies to

$$\frac{\partial C(\mathbf{x})}{\partial x_e} = \frac{\partial \mathbf{F}}{\partial x_e} \mathbf{U} + \mathbf{F}^{\mathrm{T}} \mathbf{K}^{-1}(\mathbf{x}) \left[\frac{\partial \mathbf{F}}{\partial x_e} - \frac{\partial \mathbf{K}(\mathbf{x})}{\partial x_e} \mathbf{U}(\mathbf{x}) \right].$$

Again, inserting the equality that follows from the state equation (where $\mathbf{F}^{T}\mathbf{K}^{-1} = \mathbf{U}^{T}$), the derivative of the compliance function finally reads:

$$\frac{\partial C(\mathbf{x})}{\partial x_e} = 2 \frac{\partial \mathbf{F}}{\partial x_e} \mathbf{U} - \mathbf{U}^{\mathrm{T}}(\mathbf{x}) \frac{\partial \mathbf{K}(\mathbf{x})}{\partial x_e} \mathbf{U}(\mathbf{x})$$
(A1.6)

where

$$\frac{\partial \mathbf{F}}{\partial x_e} \mathbf{U} = \mathbf{0} \,.$$

The final form of the derivation is thus:

$$\frac{\partial C(\mathbf{x})}{\partial x_e} = -\mathbf{U}^{\mathrm{T}}(\mathbf{x}) \frac{\partial \mathbf{K}(\mathbf{x})}{\partial x_e} \mathbf{U}(\mathbf{x})$$

//

The reason why the compliance function is so widely used becomes evident from Eq.(A1.5): Here, the gradient of the nested compliance function with respect to $\mathbf{U}(\mathbf{x})$ times the inverse of the stiffness matrix does not require an extra set of linear systems to be solved. If this was not the case, the linear system in Eq.(A1.3) would need to be solved once for each design variable, when using the **direct method**. For problems involving many design variables, an alternative method exists, called the **adjoint method**. In the adjoint formulation, a new set of linear systems is created from Eq.(A1.4) by introducing the vector $\boldsymbol{\lambda}_{i}$, where

$$\lambda_j = \frac{\partial \hat{C}_j(\mathbf{x}, \mathbf{U}(\mathbf{x}))}{\partial \mathbf{U}(\mathbf{x})} \mathbf{K}^{-1}(\mathbf{x}) \,.$$

Of course, here a general objective or constraint function \hat{g}_j would be inserted instead of the compliance function \hat{C}_j . Using the adjoint formulation, the linear system generally needs to be solved 'only' once for each objective or constraint function (that is a function of $\mathbf{U}(\mathbf{x})$).

APPENDIX 2 MATLAB CODES

A 55-line MATLAB code for topology optimization

```
%%%% AN 55 LINE TOPOLOGY OPTIMIZATION CODE - June 2016 - VANTYGHEM G %%%%
function top55(nelx,nely,volfrac,penal,rmin,iter)
% Example: top55(200,200,0.5,3,1.5,50)
% PREPARE FINITE ELEMENT ANALYSIS
E0=1:
Emin=1e-9;
nu=0.3;
All = [12 3 -6 -3; 3 12 3 0; -6 3 12 -3; -3 0 -3 12];
Al2 = [-6 -3 0 3; -3 -6 -3 -6; 0 -3 -6 3; 3 -6 3 -6];
B11 = [-4 \ 3 \ -2 \ 9; \ 3 \ -4 \ -9 \ 4; \ -2 \ -9 \ -4 \ -3; \ 9 \ 4 \ -3 \ -4];
B12 = [2 -3 4 -9; -3 2 9 -2; 4 9 2 3; -9 -2 3 2];
KE = 1/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]);
nodenrs = reshape(1:(1+nelx)*(1+nely),1+nely,1+nelx);
edofVec = reshape(2*nodenrs(1:end-1,1:end-1)+1,nelx*nely,1);
edofMat = repmat(edofVec, 1, 8) + repmat([0 1 2*nely+[2 3 0 1] -2 -
1],nelx*nely,1);
iK = reshape(kron(edofMat,ones(8,1))',64*nelx*nely,1);
jK = reshape(kron(edofMat,ones(1,8))',64*nelx*nely,1);
% DEFINE LOADS AND SUPPORTS
F = sparse(2*(nely+1)*(nelx+1)-(nely),1,-1,2*(nely+1)*(nelx+1),1);
U = zeros(2*(nely+1)*(nelx+1),1);
fixeddofs = 1:2*(nelv+1);
alldofs = 1:2* (nely+1)* (nelx+1);
freedofs = setdiff(alldofs,fixeddofs);
% INITIALIZE ITERATION
xold = repmat(volfrac, nely, nelx);
xnew = xold;
loop = 0;
% START ITERATION
for it = 1:iter;
  loop = loop + 1;
  % FE-ANALYSIS
  sK = reshape(KE(:)*(Emin+xnew(:)'.^penal*(E0-Emin)),64*nelx*nely,1);
  K = sparse(iK, jK, sK); K = (K+K')/2;
  U(freedofs) = K(freedofs, freedofs) \F(freedofs);
  % OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
  ce = reshape(sum((U(edofMat)*KE).*U(edofMat),2),nely,nelx);
  c = sum(sum((Emin+xnew.^penal*(E0-Emin)).*ce));
  dc = -penal*(E0-Emin)*xnew.^(penal-1).*ce;
  dv = ones(nely, nelx);
  % FILTERING/MODIFICATION OF SENSITIVITIES
  dc = imgaussfilt(dc,rmin);
  % OPTIMALITY CRITERIA UPDATE OF DESIGN VARIABLES
  11 = 0; 12 = 1e9; move = 0.2;
  while (12-11)/(11+12) > 1e-3
    lmid = 0.5*(12+11);
    xnew = max(0, max(xold-move, min(1, min(xold+move, xold.*sqrt(-
dc./dv/lmid)))));
    if sum(xnew(:)) > volfrac*nelx*nely, 11 = lmid; else 12 = lmid; end
  end
  change = max(abs(xnew(:)-xold(:)));
  xold = xnew;
  % PRINT RESULTS
  fprintf(' It.:%5i Obj.:%11.4f Vol.:%7.3f ch.:%7.3f\n',loop,c, ...
   mean(xnew(:)), change);
  % PLOT DESIGN VARIABLES
  colormap(gray); imagesc(1-xnew); caxis([0 1]); axis equal; axis off;
drawnow;
end
```

A novel material interpolation scheme for topology optimization with improved

mechanical and thermal properties

%%%%% TOPOLOGY OPTIMIZATION / Thermal + Structural / CODE NOV, 2018 %%%% %%%%% A Novel Material Interpolation Scheme for Topology Optimization %%%% with Improved Mechanical and Thermal Properties *** 응응응응 **** Vantyghem G - De Corte W - Steeman M - Boel V 2222 addpath('C:\Users\...\MMAscripts'); % PARAMETERS nelx = 120; = 600; nelv = nelx*nely; nele ini = 0.5; = 4; rmin = 300; niter = 0.5; Vmax % MATERIAL PROPERTIES Emax = 2500; kmax = 0.275: Emin = 0.001;kmin = 0.025;% PREPARE FEA - Structural S.nu = 1/3; S.A11 = [12 3 -6 -3; 3 12 3 0; -6 3 12 -3; -3 0 -3 12]; S.A12 = [-6 -3 0 3; -3 -6 -3 -6; 0 -3 -6 3; 3 -6 3 -6]; S.B11 = [-4 3 -2 9; 3 -4 -9 4; -2 -9 -4 -3; 9 4 -3 -4]; S.B12 = [2 -3 4 -9; -3 2 9 -2; 4 9 2 3; -9 -2 3 2]; S.KE = 1/(1-S.nu^2)/24*([S.A11 S.A12;S.A12' S.A11]+S.nu*[S.B11 S.B12; S.B12' S.B11]); S.nodenrs = reshape(1:(1+nelx)*(1+nely),1+nely,1+nelx); S.edofVec = reshape(2*S.nodenrs(1:end-1,1:end-1)+1,nelx*nely,1); S.edofMat = repmat(S.edofVec, 1, 8) + repmat([0 1 2*nely+[2 3 0 1] -2 -1],nelx*nelv,1); = reshape(kron(S.edofMat,ones(8,1))',64*nelx*nelv,1); SiK S.jK = reshape(kron(S.edofMat,ones(1,8))',64*nelx*nely,1); S.maxdof = 2* (nely+1)* (nelx+1); % PREPARE FEA - Thermal T.A1 = [8 -2; -2 8];T.A2 = [-4 -2; -2 -4];T.KE = 1/12 * [T.A1 T.A2; T.A2 T.A1]; T.nodenrs = reshape (1: (nely+1)*(nelx+1), nely+1, nelx+1); T.nodeids = reshape(T.nodenrs(1:end-1,1:end-1),nely*nelx,1); T.edofVec = T.nodeids(:)+1; T.edofMat = repmat(T.edofVec,1,4)+repmat([0 nely+[1 0] -1],nelx*nely ,1); T.iK = reshape(kron(T.edofMat,ones(4,1))',16*nele,1); T.iK = reshape(kron(T.edofMat,ones(1,4))',16*nele,1); T.maxdof = (nely+1)*(nelx+1); % DEFINE LOADS AND SUPPORTS - Structural S.F = sparse([1:2:2*(nely+1)-1 S.maxdof-2*(nely+1)+1:2:S.maxdof-1],1,-1,S.maxdof,1); S.U = zeros(S.maxdof, 1); S.fixeddofs = 2:2*(nely+1):nelx*2*(nely+1); S.fixeddofs = [S.fixeddofs 2*(nely+1)-1]; S.alldofs = 1:S.maxdof; S.freedofs = setdiff(S.alldofs,S.fixeddofs); % DEFINE LOADS AND SUPPORTS - Thermal T.F = sparse(T.maxdof-(nely+1)+1:T.maxdof,1,-1,T.maxdof,1); T.U = zeros(T.maxdof, 1);T.fixeddofs = 1:nely+1; T.alldofs = 1:T.maxdof; T.freedofs = setdiff(T.alldofs,T.fixeddofs); % PREPARE FILTER iH = ones(nelx*nely*(2*(ceil(rmin)-1)+1)^2,1); iH = ones(size(iH)): sH = zeros(size(iH));

```
k = 0;
for i1 = 1:nelx
  for j1 = 1:nely
    e1 = (i1-1)*nelv+j1;
    for i2 = max(i1-(ceil(rmin)-1),1):min(i1+(ceil(rmin)-1),nelx)
      for j2 = max(j1-(ceil(rmin)-1), 1):min(j1+(ceil(rmin)-1), nely)
        e2 = (i2-1) *nely+j2;
        k = k+1;
        iH(k) = e1;
        jH(k) = e2;
        sH(k) = max(0, rmin-sqrt((i1-i2)^{2}+(j1-j2)^{2}));
      end
    end
  end
end
H = sparse(iH, jH, sH);
Hs = sum(H, 2);
% INITIALIZE ITERATION
xnew = repmat(ini,nely,nelx);
xold2 = xnew(:); xold1 = xnew(:); xval = xnew(:);
                                     = xmax;
xmax = ones(nele,1);
                                upp
xmin = zeros(nele,1);
                                100
                                      = xmin;
% START ITERATION
for iter = 1:niter
  % FEA Structural
  S.sK = reshape(S.KE(:)*(Emin+xnew(:)'./(3.6-2.6*xnew(:)')*(Emax-
Emin)),64*nelx*nely,1);
  S.K = sparse(S.iK,S.jK,S.sK); S.K = (S.K+S.K')/2;
  S.U(S.freedofs) = S.K(S.freedofs, S.freedofs) \ (S.F(S.freedofs) -
S.K(S.freedofs, S.fixeddofs) *S.U(S.fixeddofs));
  % FEA Thermal
  T.sK =
reshape(T.KE(:)*(kmin+(0.787*xnew(:)'.^2+0.136*xnew(:)'+0.078*xnew(:)'.^-1
)*(kmax-kmin)),16*nelx*nelv,1);
  T.K = sparse(T.iK,T.jK,T.sK); T.K = (T.K+T.K')/2;
  T.U(T.freedofs) = T.K(T.freedofs, T.freedofs) \ (T.F(T.freedofs) -
T.K(T.freedofs, T.fixeddofs) *T.U(T.fixeddofs));
  % OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
    v = mean(xnew(:));
   dv = ones(nele, 1) /nele;
  S.ce = reshape(sum((S.U(S.edofMat)*S.KE).*S.U(S.edofMat),2),nely,nelx);
  S.c = sum(sum((Emin+xnew./(3.6-2.6*xnew)*(Emax-Emin)).*S.ce));
  S.dc = -1*(Emax-Emin)*3.6./(3.6-2.6*xnew).^2.*S.ce;
  T.ce = reshape(sum((T.U(T.edofMat)*T.KE).*T.U(T.edofMat),2),nely,nelx);
  T.c = sum(sum((kmin+(0.787*xnew.^2+0.136*xnew+0.078*xnew.^-1)*(kmax-
kmin)).*T.ce));
  T.dc = -1*(kmax-kmin)*(2*0.787*xnew+0.136-0.078*xnew.^{-2}).*T.ce;
  f1 = 3000;
  f2 = 3000;
  obj = S.c/f1-T.c/f2;
  % FILTERING/MODIFICATION OF SENSITIVITIES
  S.dc(:) = H*(S.dc(:)./Hs);
  T.dc(:) = H*(T.dc(:)./Hs);
  % MMA UPDATE OF DESIGN VARIABLES
      = 1;
  m
  n
       = nele;
  f0val = obj;
  df0dx = S.dc(:)/f1-T.dc(:)/f2;
  fval = v/Vmax-1;
  dfdx = dv(:) '/Vmax;
  a0 = 1; a1 = zeros(m,1); c1 = 1000*ones(m,1); d1 = zeros(m,1);
  [xmma, ymma, zmma, lam, xsi, eta, mu, zet, s, low, upp] = ...
mmasub(m,n,iter,xval,xmin,xmax,xold1,xold2,f0val,df0dx,fval,dfdx,low,upp,a
0,a1,c1,d1);
  xold2 = xold1; xold1 = xval; xval = xmma;
```

```
change = max(abs(xval(:)-xold1(:)));
xnew = reshape(xmma,nely,nelx);
xnew(:) = (H*xnew(:))./Hs;
% PRINT RESULTS
fprintf([' I:%5i Obj:%8.2f Str.compl:%8.2f Therm.compl:%8.2f '...
 ' Volume:%5.2f Change:%6.3f\n'],iter,obj,S.c/f1,T.c/f2,v,change);
% PLOT DESIGN VARIABLES
figure(1);
set(gcf,'position',[100 240 1200 300]);
colormap(jet); imagesc(rot90(xnew)); caxis([0 1]); axis equal; axis
tight; axis off;
colorbar('eastoutside'); drawnow;
end
```

Topology optimization with stress constraints (Bresler Pister)

```
** TOPOLOGY OPTIMIZATION WITH STRESS CONSTRAINTS - Jun 2018 - VANTYGHEM **
= 40:
nelx
                                      % number of elements in x-dir
       = 20;
nelv
                                      % number of elements in v-dir
dikte
      = 80;
nele
       = nelx*nely;
                                      % number of finite elements
                                      % degrees of freedom
ndofs
       = 2*(nely+1)*(nelx+1);
ini
       = 0.5;
                                      % initial density
                                      % penalization factor p
p
       = 3;
       _
          2.8;
                                      % relaxation parameter g
α
rmin
       = 1.5:
                                      % filter radius
niter
       = 600;
                                      % number of iterations
       = 16:
                                      % max tensile strength
Smax
% PREPARE FINITE ELEMENT ANALYSIS
E0 = 30000;
                                      % Young's modulus solid (MPa)
                                      % Young's modulus void (MPa)
Emin = 1e-3;
nu = 0.25;
                                      % Poisson factor
A11 = [12 3 -6 -3; 3 12 3 0; -6 3 12 -3; -3 0 -3 12];
A12 = [-6 - 3 \ 0 \ 3; -3 - 6 - 3 - 6; \ 0 - 3 - 6 \ 3; \ 3 - 6 \ 3 - 6];
B11 = [-4 3 -2 9; 3 -4 -9 4; -2 -9 -4 -3; 9 4 -3 -4];
B12 = [ 2 -3 4 -9; -3 2 9 -2; 4 9 2 3; -9 -2 3 21;
KE = dikte/(1-nu^2)/24*([A11 A12;A12' A11]+nu*[B11 B12;B12' B11]);
nodenrs = reshape(1:(1+nelx)*(1+nely),1+nely,1+nelx);
edofVec = reshape(2*nodenrs(1:end-1,1:end-1)+1,nele,1);
edofMat = repmat(edofVec,1,8)+repmat([0 1 2*nely+[2 3 0 1] -2 -1],nele,1);
iK = reshape(kron(edofMat,ones(8,1))',64*nele,1);
jK = reshape(kron(edofMat,ones(1,8))',64*nele,1);
% DEFINE LOADS AND SUPPORTS
load = [2 4 6 8]; loads = [load load+(nely+1)*2 load+(nely+1)*4];
F = sparse(loads, 1, -125, ndofs, 1);
                                  % load vector (N)
U = zeros(ndofs, 1);
                                      % displacement Vector (mm)
symm = 1:2:2*(nely+1);
fixeddofs = [symm];
                                      % supports and symmetry
alldofs = 1:ndofs;
                                      % all degrees of freedom
freedofs = setdiff(alldofs,fixeddofs);% free degrees of freedom
% INITIALIZE ITERATION and PREALLOCATION of MEMORY
xnew = repmat(ini,nely,nelx);
xold2 = xnew(:) ; xold1 = xold2 ; xvv
xmax = ones(nele,1) ; xmin = 0.1*ones(nele,1);
                                          ; xval = xold1;
low
    = ones(nele,1) ; upp = ones(nele,1) ;
                    ; J2 = ones(nele,1)
T1 = ones(nele, 1)
                                           ; Bresler = ones(nele,1);
B = [0.5, 0; 0, -0.5; -0.5, 0.5]; B = [-abs(B), B, abs(B), -B];
D = E0/(1-nu^2) * [1 nu 0]
                     1
                nu
                          Ω
                0
                     0 (1-nu)/2];
D1 = E0/(1-nu) * [1 1 0];
                                            % D1 matrix
D2 = (E0/(1-nu^2))^2 \dots
                                            % D2 matrix
          [ nu^2-nu+1
                             (-nu^2+4*nu-1)/2
                                                   Ω
                              nu^2-nu+1
            (-nu^2+4*nu-1)/2
                                                   0
                 0
                                              3*((1-nu)/2)^2];
                                    0
st = Smax;
   = Smax;
SC
sb = sc*1.16;
c11 = (st-sc)/(st+sc) * (4*sb^2-sb*(sc+st)+sc*st) / (4*sb^2+2*sb*(st-sc)-
sc*st);
```

```
c22 =
         1 /(st+sc) * (sb*(3*st-sc)-2*sc*st) / (4*sb^2+2*sb*(st-sc)-
sc*st);
c00 = c11*sc-c22*sc^2;
ss = sc/st;
% PREPARE FILTER
iH = ones(nelx*nely*(2*(ceil(rmin)-1)+1)^2,1);
jH = ones(size(iH));
sH = zeros(size(iH));
k = 0;
for i1 = 1:nelx
  for j1 = 1:nely
    e1 = (i1-1)*nely+j1;
    for i2 = max(i1-(ceil(rmin)-1),1):min(i1+(ceil(rmin)-1),nelx)
      for j2 = max(j1-(ceil(rmin)-1),1):min(j1+(ceil(rmin)-1),nely)
        e2 = (i2-1)*nely+j2;
        k = k+1:
        iH(k) = e1;
        jH(k) = e2;
        sH(k) = max(0, rmin-sqrt((i1-i2)^2+(j1-j2)^2));
      end
    end
 end
end
H = sparse(iH, jH, sH);
Hs = sum(H, 2);
% START ITERATION
for iter = 1:niter
tic
   if iter < 11
    threshold = 0.65+0.2*(iter-1)/10;
   else
   threshold = 0.85;
   end
sK = reshape(KE(:)*(Emin+xnew(:)'.^p*(E0-Emin)),64*nele,1);
K = sparse(iK, jK, sK); K = (K+K')/2;
% SPRING SUPPORTS
spring = -lel;
K(ndofs-4*(nely+1), ndofs-4*(nely+1)) = K(ndofs-4*(nely+1), ndofs-
4*(nely+1))+spring;
K(ndofs-2*(nely+1), ndofs-2*(nely+1)) = K(ndofs-2*(nely+1), ndofs-2*(nely+1))
2*(nely+1))+spring;
K(ndofs,ndofs) = K(ndofs,ndofs)+spring;
% SOLVE U (FREE DOFS)
U(freedofs) = K(freedofs, freedofs) \F(freedofs); % solving state equation
% STRESS CALCULATION
I1 = U(edofMat)*(D1*B)';
J2 = sum(U(edofMat)*B'*D2*B.*U(edofMat),2);
Bresler = reshape(1/ss*(sqrt(J2)-c00-c11*I1-c22*I1.^2),nely,nelx);
stress = Bresler;%.*xnew.^p;
                                      % penalized element stresses
% PLOT FIGURE
figure(1); clf('reset'); scale = 100; % Deformation Scale Factor
xx = zeros(4, nele);
yy = zeros(4, nele);
iii=0;
for ely = 1:nely
    for elx = 1:nelx
      iii=iii+1;
      n1 = (nely+1) * (elx-1) +ely;
      n2 = (nely+1) * elx + ely;
      Ue = scale*U([2*n1-1;2*n1; 2*n2-1;2*n2; 2*n2+1;2*n2+2;
2*n1+1;2*n1+2],1);
```

```
ly = ely-1; lx = elx-1;
      xx(:,iii) = [Ue(1,1)+lx Ue(3,1)+lx+1 Ue(5,1)+lx+1 Ue(7,1)+lx ]';
      yy(:,iii) = [Ue(2,1)-ly Ue(4,1)-ly Ue(6,1)-ly-1 Ue(8,1)-ly-1]';
    end
end
cc = (stress)'.*xnew'.^(p);
patch(xx, vy, cc(:)', 'EdgeColor', 'k');
colormap(jet); caxis([0 Smax]); axis equal; axis off;
colorbar('eastoutside'); drawnow;
% FIND HIGHEST STRESSED ELEMENTS
[stressB, stressI] = sort(stress(:), 'descend');
count = find(stressB>(Smax*threshold)); nsconstr = max(count);
 if isempty (nsconstr)
   nsconstr = 1:
end
LocalList = stressI(1:nsconstr); % find index of most stressed elements
% FE-ANALYSIS PSEUDO LOADS
Ukrul = zeros(2*(nely+1)*(nelx+1), nsconstr);
parfor j = 1:nsconstr
  e = LocalList(j);
  Ue = U(edofMat(e,:));
  Loadps = (1/ss)*((Ue'*B'*D2*B*Ue).^(-1/2)*B'*D2*B*Ue-(c11*D1*B)'-
(2*c22*D1*B*Ue*D1*B)');
  Fkrul = sparse(edofMat(e,:),1,Loadps',ndofs,1);
  Ukrul(freedofs,j) = K(freedofs,freedofs)\Fkrul(freedofs);
end
% OBJECTIVE FUNCTION AND SENSITIVITY ANALYSIS
   sig = xnew(LocalList).^(p-g).*stress(LocalList);
  dsig = zeros(nsconstr,nele);
  for e = 1:nsconstr
    Uk = Ukrul(:,e);
    sige = sum((Uk(edofMat)*KE).*U(edofMat),2);
    dsig(e,:) = -p^*(E0-Emin)^*xnew(:)'.^(p-
1).*sige'.*xnew(LocalList(e))^(p-g);
  end
  krondelta = sparse(1:nsconstr,LocalList',(p-q)*xnew(LocalList)'.^(p-q-
1).*stress(LocalList)',nsconstr,nele);
  dsig = (dsig+krondelta);
  % FILTERING/MODIFICATION OF SENSITIVITIES
dv
   = H*(dv./Hs);
  for e = 1:nsconstr
                                      % filtering all dsig
    test = dsig(e,:)';
    test = H*(test./Hs);
   dsig(e,:) = test';
  end
 % MMA UPDATE OF DESIGN VARIABLES
        = nsconstr; % The number of constraints
  m
        = nele:
                      % The number of variables
 n
  f0val = v/nele;
                     % The value of the objective function
  df0dx = dv/nele;
                     % Column vector with the derivatives of the obj.
  fval = sig/Smax-1; % Column vector with the values of the constraint f
 dfdx = dsig/Smax; % (m x n)-matrix with the derivatives of the constr
  a0 = 1; a1 = zeros(m,1); c1 = 100000*ones(m,1); d1 = ones(m,1);
  [xmma, ymma, zmma, lam, xsi, eta, mu, zet, s, low, upp] = mmasub(m, n, iter, ...
  xval,xmin,xmax,xold1,xold2,f0val,df0dx,fval,dfdx,low,upp,a0,a1,c1,d1);
 xold2 = xold1; xold1 = xval; xval = xmma;
```

```
change = max(abs(xval(:)-xold1(:)));
xnew = reshape(xmma,nely,nelx);
xnew(:) = (H*xnew(:))./Hs;
figure(2); set(gcf,'position',[50 450 630 300])
colormap(gray); imagesc(1-xnew); caxis([0 1]); axis equal; axis off;
drawnow;
% PRINT RESULTS
time = toc;
fprintf('I:%5i Vol:%1.3f Stress:%7.2f nsconstr:%7.f
Time:%7.f\n'...
,iter,v/nele,max(sig),nsconstr,time);
end
```
APPENDIX 3 T-SECTION VS TOPOLOGY OPTIMIZED GIRDER

In this appendix, the topology optimized girder is numerically compared to a T-section girder with the same flange size (585 mm by 97 mm) and the same overall depth (389 mm) (see Figure A3.1).

First, the mid-span deflections of the topology optimized shape are obtained through numerical calculation in Abaqus. The material density (ρ) is set to 2500 kg/m³, the young's modulus (*E*) is 32800 N/mm², and the ratio of Poisson (ν) is 0.24. The values for the defection are respectively 0.29 mm under self-weight (615 kg) and 0.27 mm under the life loads (540 kg), both measured at the top of the girder. The numerical results are presented in Figure A3.2, indicating a total deflection of 0.56 mm.

Secondly, the width of the web of the T-section girder is determined using Eq.(A3.1) where the maximum deflection of the T-section girder is set equal to the result by the numerical study. The formulation reads:

$$\Delta_{\max} (\text{at center}) = \frac{5}{384} \frac{wl^4}{EI}.$$
(A3.1)

with the uniformly distributed load (w) evaluated as 1.77 kN/m (self-weight) + 1.32 kN/m (life load), the girder's length (l) is 4 m, and its moment of inertia is found to be (I) 556024703 mm⁴, corresponding to a web thickness of 47 mm.



Figure A3.1. Rhino model of the topology optimized girder.



Figure A3.2. The results of from the numerical study of the topology optimized girder showing a) the deformation under self-weight and b) deformations under the testing load.

The resulting volume of the T-section concrete girder is 720 kg, nearly 20% higher than that of the optimized beam (615 kg), hence clarifying the incentive to employ topology optimization and manufacture complex geometries. The width of the T-section's web is purely theoretical and does not comply to any rules related to minimum width. In reality, the difference in weight is therefore larger. Furthermore, for larger bridge structures, where the self-weight component becomes even more significant, additional savings can be expected.

